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Mesh Generation: A Critical Review

Lokesh Singh

Mechanical Engineering Department, Lokeshsingh@gmx.com
GD Rungta College of Engineering & Technology, Bhilai, 490023, India

Abstract

Meshing is a process of spatial decomposition. With the required topology and geometric constraints a physical space is decomposed in small parts (called as elements). Mesh generation is a relatively young and multidisciplinary science, it requires expertise in the areas of numerical analysis, meshing algorithms, geometric design, computational geometry, computational physics, scientific visualization and software engineering to create a mesh tool. Meshing can be used for a wide variety of applications; the principal application of interest is the finite element method. In this study, an overview of mesh generation process as well as history, mesh quality and their types, meshing types by configuration & element wise and application of meshing are discussed.

Keywords: Mesh, Mesh quality, Mesh types.

1. Introduction

Meshing is a process of spatial decomposition. With the required topology and geometric constraints a physical space is decomposed in small parts (called as elements), it may also be termed as the process of breaking up a physical domain into smaller sub-domains (Lu et al. 2001). Mesh generation is a relatively young and multidisciplinary science, it requires expertise in the areas of numerical analysis, meshing algorithms, geometric design, computational geometry, computational physics, scientific visualization and software engineering to create a mesh tool. Considerable progress has been made in meshing technology; there are wide variety of tools available commercially and open sources (Hansen and Owen 2008). Over the last 30 years finite element analysis has evolved hand in hand with the ever increasing hardware capability. With the advent of the fast advanced solvers, meshing has been and remains to be significant bottleneck in fully exploiting FEA's great potential (Canann et al. 1997). Even with the fully automatic mesh generators there are many cases where the solution time can be less than the meshing time. Although meshing can be used for a wide variety of applications, the principal application of interest is the finite element method. Surface domains may be subdivided into triangular or quadrilateral shapes, while volumes may be subdivided primarily into tetrahedral or hexahedral shapes. Meshing algorithms ideally define the shape and distribution of the elements.
Mesh generation is usually considered as the pre-processing step for numerical computational techniques.

2. A Brief history of FEM and Meshing

In earlier forties and fifties finite element method was pioneered by engineers and practitioners. Then during sixties, mathematician established the theoretical and mathematical foundation of FEM (George et al. 2007). The FEM widely used in various category of peoples and in the year 1972, about 1004 papers related to FEM were published (Remacle 2009). Afterward a number of applications in different fields of engineering motivated researchers to develop various mesh generation methods. Except where the mesh generation is straight forward like square region, many applications in arbitrary domain and arbitrary material variation require designing and implementing mesh generation methods. In 1970s George (A.EI-hamalawi 2004) Introduced an advancing front method where as Delaunay Triangulation was introduced by Hermeline and Watson (Legrand et al. 2000). Same year, An article is published where the Delaunay triangulation and advancing front mesh generation algorithm is clearly described for FEM by C.O Fredrick et.al entitled " Two Dimensional automatic mesh generation for

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structural analysis" in the international journal for numerical methods in engineering (Cheng et al. 2012). Afterwards many mesh generation algorithms were developed by various researchers.

3. Mesh Quality

In general, poor quality elements are those elements with one (or more) of the following & also shown in Figure 1.

(a) Ratio of maximum side length to minimum side length is larger than 10.
(b) Minimum interior angle is smaller than 20^0.
(c) Maximum interior angle is larger than 120^0.

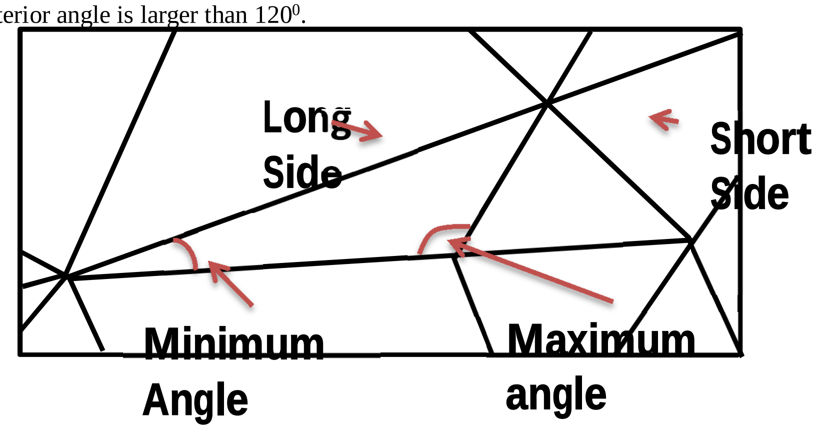

Figure 1: Triangular mesh element showing the longest side, shortest side, maximum interior angle and the minimum interior angle.

The mesh quality can have a considerable impact on the computational analysis in terms of the quality of the solution and the time needed to obtain it. This aspect becomes especially important if poorly conditioned problems, non-linear, and/or transient analysis are considered. From this point of view, the evaluation of the quality of the mesh is very useful because it provides some indication of how suitable a particular discretization is for the analysis type under consideration.
To compute the quality of individual elements and how to quantify the overall quality of a mesh there are following ways (Andre Bakker 2006), (G. Lee 2000):

Aspect ratio:
Aspect ratio= Maximum element edge length / Minimum element edge length
Ideal value = 1 (Acceptable < 5).

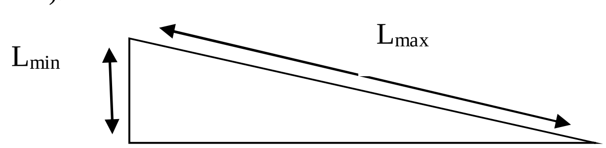

Figure 2: Minimum and Maximum length of element

Skew:
Ideal value=0 (Acceptable<45^0)
Skew for quadrilateral element=90^0 minus the minimum angle between the two lines joining the opposite mid-side of the element (a)

Figure 3: Skew for Quadrilateral and Triangle element

Skew for triangular element=90^0 minus the minimum angle between the lines from each node to the opposing mid-side and between the two adjacent mid-sides at each node of the element.

Jacobian:
Ideal value = 1.0 (Acceptable > 0.6)
In simple terms, the Jacobean is a scale factor arising because of the transformation of the coordinate system. Elements are transformed from the global coordinates to local coordinates (defined at the centroid of every element) for faster analysis times.

Stretch:
Ideal value: 1.0 (Acceptable>0.2)
For quadrilateral element stretch $= L_{min} * \sqrt{2}/d_{max}$
Stretch for triangular element $= R * \sqrt{12}/L_{max}$

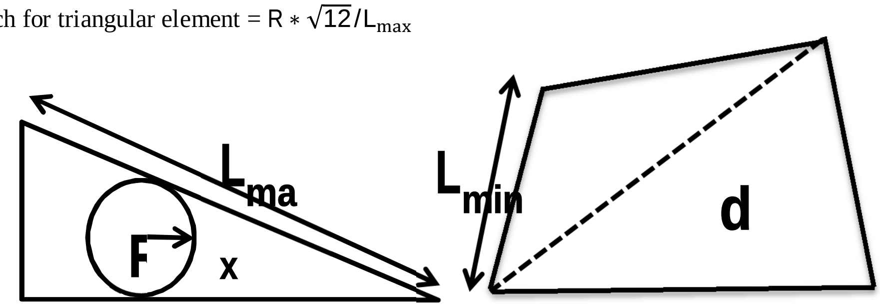

Figure 4: Stretch for Triangle and quadrilateral element

Included angles:
Included angle check is applied for individual interior angles of element
Quadrilateral ideal value= 90^0 (Acceptable=$45^0 < \theta < 135^0$)
Triangular ideal value= 60^0 (Acceptable =$20^0 < \theta < 120^0$)

Minimum element length:
This is a very important check for crash analysis (time step calculations). It is also applied in general to check for the minimum feature length captured and the presence of any zero length elements.

4. Meshing Types by Configuration

Meshes can be categorized as structured, unstructured and hybrid meshes by configuration. The choice of the mesh type is clearly related to the application.

4.1 Structured Meshes

Structured meshes are identified by regular connectivity as shown in Figure 5. This model is highly space efficient, i.e. since the neighborhood relationships are defined by storage arrangement. The possible element choices are quadrilateral in 2D and hexahedra in 3D.

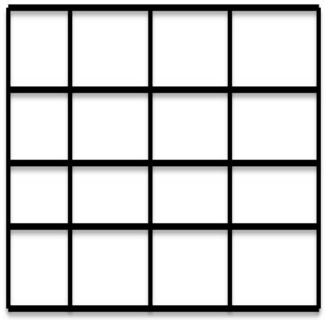

Figure 5: Structured Mesh

4.2 Unstructured Meshes

An unstructured mesh is identified by irregular connectivity as shown in Figure 6. It cannot easily be expressed as two-dimensional or three-dimensional arrays in computer memory. This allows for any possible element that a solver might be able to use. Compared to structured meshes, this model can be highly space inefficient since it calls for explicit storage of neighborhood relationships. These grids typically employ triangles in 2D and tetrahedral in 3D.

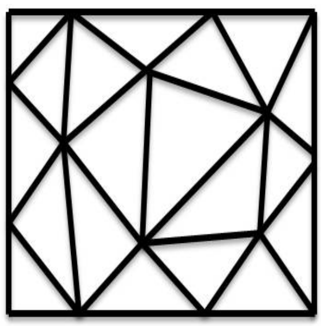

Figure 6: Unstructured Mesh

In Table 1(Liu et al. 2004), (Tautges 2004), (Molnar 2014). A brief comparison is shown in between structured and unstructured mesh

Table Error! No text of specified style in document.**:** Comparison table of Merits/Demerits of Structured Mesh and Unstructured Mesh

	Structured mesh	**Unstructured mesh**
Labor intensive	Low	High
Time consuming	Low	High
Mesh modification	No	Yes
Accuracy	Defined	undefined
Handling complex geometry	Difficult	Relatively easier
Automation	High	Low

4.3 Hybrid Meshes

A hybrid mesh is formed by a number of structured meshes arranged in an overall unstructured pattern. Hybrid meshes fall somewhere in between structured and unstructured meshes as shown in Figure 7. These meshes are used in problems with complicated geometries.

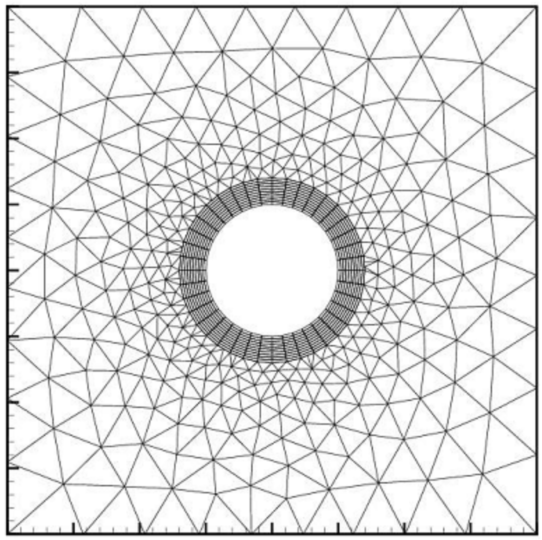

Figure 7: Hybrid mesh (Hannoun's 2015)

5. Meshing Types by Elements

Meshes can be divided into two main groups by elements. These are Triangle/ Tetrahedral and Quadrilateral/ Hexahedral meshes, which are considered in two-dimension / three-dimension, respectively.

5.1 Tri / Tetrahedral Meshes

Triangle and tetrahedral meshes are the most common forms of unstructured mesh generation. Most techniques currently in use can be considered in three main categories: Octree, Delaunay and Advancing Front techniques. Tri / tetrahedral meshes can also be constructed from quad / hexahedral mesh elements.

5.2 Quad / Hexahedral Meshes

Surface domains can be subdivided into quadrilateral elements, whereas volumes can be subdivided into hexahedral elements by structured as well as unstructured meshing methods. Isoparametric coordinates can be used to generate both quad / hexahedral meshes, which are considered as structured. As for unstructured quad / hexahedral meshes, they are generated using direct and indirect approaches (quadrilateral mesh directly generates from mesh generation methods that called direct approach whereas indirect approach means combining triangles into quadrilateral and leads to related merging). Meanwhile, some methods are also available that combine hexahedral and Tetrahedral elements in a single three-dimensional domain.

Generally, unstructured mesh generation algorithms use triangle and tetrahedral mesh elements. As a result of this, most of the literature and software use triangle and tetrahedral elements. There is a significant group of literature that focuses on unstructured quad and hexahedral methods.

In Table 2 (Li and Cheng 2000), (Egidi and Maponi 2008), (H. Zhang et al. 2007), (Benzley et al. 1995), (Kraft 1999). A brief comparison is shown between triangle / tetrahedral and quadrilateral / hexahedral mesh.

Table 2: Comparison chart for triangle/tetrahedral and quadrilateral/hexahedral mesh

	Triangle/Tetrahedral	**Quadrilateral/Hexahedral**
Mesh generation	Easy to generate or less complex	Harder to generate
Accuracy of result	Less accurate analysis result	More accurate analysis result
Element versus node density	More elements	Half element as compare to triangle elements
Cost of FEM simulation	More cost of FEM simulation	Less cost of FEM simulation
Rigid geometric objects	Less efficient numerical method for generating triangular grid	More efficient numerical method for generating triangular grid
Element require for accuracy	4 to 10 times more element than quad/hex elements required for accuracy	Less number of elements requirement
In case of element destroyed (i.e. distortion created by large deformation FEA)	Result gives critical error	Results produce detrimental displacement field

6. Application Area of Meshing

The mesh generation is an important part of the finite element analysis and the solution obtained with the finite element method depends strongly on the quality of the mesh. A good meshing method must generate a correct mesh, i.e. boundary conforming, without holes, without free edges and without intersecting elements and containing as few badly shaped elements as possible. The accuracy of FEM computational depends on the mesh size and the distribution of mesh points. Mesh generation is critical step in wide range of engineering practices, such as the study of scientific simulation by FEA, of design automation with geometrical modelling and of data representation with graphics and visualization. Few general application of meshing in different areas is as follows:

(a) It is the critical step in wide range of engineering practice such as the study of scientific simulation of FEA, design automation with geometrical modelling and data representation with graphic and visualization.
(b) The metal forming industry has widely accepted finite element simulation as an efficient tool for designing and analysis their manufacturing process.
(c) Meshing is also used for computational method to simulate mechanical behaviour of poly-crystals.
(d) A polygon meshes extensively used in computer graphics for ray tracing and collision detection and visualization.
(e) Mesh generation is used in photogrammetric survey in Archaeology in GIS.
(f) Mesh generation is also used in fluid dynamic analysis of the coolant behaviour inside the reactor core and also seeing increased use in active research areas such as computational medicines and computational biology.
(g) FEA mesh is commonly used for orthopaedic implant design, implant structure interaction, bone re-modelling, and joint contact mechanics and also used for preoperative planning.

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