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Finite Element Analysis of 2D Heterogeneous Objects Using Material Based Graded Elements

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ABSTRACT

A heterogeneous object is referred to a solid object made of different constituent materials. The object is made up of finite collection of regions having set of prescribed material classes of continuously or discontinuously varying material properties. These properties have an abrupt or gradual change across the interference of the material regions. Application of heterogeneous material range from bio-material products, aerospace structures, to meta-material with demonstrated performance improvement in structural, mechanical, electronics and or sensing property in these application , the performance goal are achieved because of the capability of varying material properties globally across the spatial domain. In order to take the full advantage of the greatest potential of heterogeneous object, we must go for developing their computer modelling and analysis techniques.

Based upon the computerized object model, finite element methods are often used to analyse these objects. In homogeneous objects, such analysis typically involves lengthy analysis model setup time, i.e. material property, mesh construction and mesh refinement, boundary condition setup and so on. It is estimated that analysis model setup takes more than 80% of the overall analysis time. In the context of heterogeneous object analysis, this issue is further exacerbated. In case of heterogeneous objects each and every point in the domain material property is vary (i.e. young modulus and Poisson ratio) so for this type of material for getting exact result we define new type of elements (called as Material Based Graded Element) in the domain with the help of this we can come closer to the exact result and also find the difference. The objective of the paper is to get near to accurate result of heterogeneous object by using Material-Based Graded elements.

In this paper we have considered linear and nonlinear material variation with two approaches average and integration approach for 2D and four types of graded element, Average Young's modulus of domain, Average Young's modulus of element, Bi-Linear variation of Young's modulus along X & Y direction, Bi-Non linear variation of Young's modulus along X & Y direction for 2D .

Keywords**:** Heterogeneous Objects, Material-Based Graded elements, Young's modulus, Linear, Non-linear.

1. INTRODUCTION

The term heterogeneous object refers to the non-uniformity of the material composition of objects as shown in (Figure 1) having two different Material. No matter whether it is homogeneous or heterogeneous object its physical properties are influenced by its micro and macro structure significantly therefore a practical and realistic heterogeneous model is heterogeneous in both material composition and structure.

Vol.4 Issue 5, May 2016 Pgs: 130-139

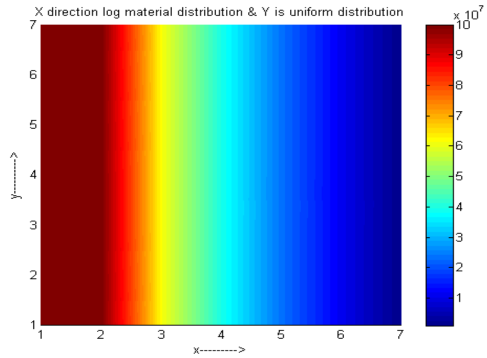

Figure 1: Model heterogeneous object

Heterogeneous objects are generally classified as objects with clear domain and those with continuous material variation. Current development of modelling system is concern with representing and controlling material composition within object boundary [8-9]. Heterogeneous objects have gained great popularities and research interest in the past few decades. Different from traditional parts which are made of the same uniform material (e.g. like copper cylinder or ceramics pistons).Heterogeneous objects refer to object with spatially different material composition or structures. Multi material objects embedded sensors /actuators, objects with functionally graded material (FGM) distributions, micro-electro-mechanical (MEMS) device, and porous structures and composite are typical examples of heterogeneous objects [10].

Based upon the computerized object model, finite element methods are often used to analyse these objects. In homogeneous objects, such analysis typically involves lengthy analysis model setup time, i.e. material property, mesh construction and mesh refinement, boundary condition setup and so on. It is estimated that analysis model setup takes more than 80% of the overall analysis time. In the context of heterogeneous object analysis, this issue is further exacerbated. In case of heterogeneous objects each and every point in the domain material property is vary (i.e. *young modulus and Poisson ratio)* so for this type of material for getting exact result we define new type of elements (called as Material Based Graded Element) in the domain with the help of this we can come closer to the exact result and also find the difference.

2. Literature Review

Heterogeneous objects have been analysed by both theoretically and experimentally by many researchers and they have worked to optimize the various parameters such as material property, meshing etc.

In order to have some insight into present state of knowledge, extensive literature survey has been conducted. An attempt has also been made to highlight the gaps in the knowledge regarding to the material property assigning in the heterogeneous object and analysis of heterogeneous object which in turn has help to decide the scope of the present study. Two-dimensional computer simulation of deformation process in models of heterogeneous materials analysis and description of the behaviour of composite and porous materials under impact loading [1]. A numerical method reported for Analysis of two different kinds of heterogeneous elastic multi-layered cylinder with different value for both elastic modulus and poison ratio and another is an elastic hollow cylinder with continuously grade material properties [2]. The behaviour of many of the heterogeneous structural material is submitted like concrete, has different behaviour under tensile stresses in comparison to their behaviour under compressive stresses [3].

The aim of the author was to interpret behaviour of such materials subjected to tensile stress, by using newly introduce concept of fractal geometry. A Comparison is presented between subject-specific finite element analyses (FEA) of a human femur to experimental measurements, using different methods for assigning material property to the FE model [4]. An integrated design and analysis approach for heterogeneous object realization, which employs a unified design and analysis model based on B-spline representation and allows for direct interaction between the design and analysis model without laborious meshing operation. In the design module, a new approach for intuitively modeling of multi-material objects, termed heterogeneous lofting, is presented [7].

Two specific patient finite element model were create and analysis under the compressive load. Vertebral bone was treated as an orthotropic material. Commercial software for medical image processing (CT imaging)

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Vol.4 Issue 5,
May 2016
Pgs: 130-139

and FE analysis ANSIS were used for mechanical analysis of the FE model [3]. B-spline surface represent material composition solid model of proximal femur graded element is used to defining inhomogeneous isotropic elastic property in the finite element model to improve the performance. Although there are many heterogeneous object modelling and analysis schemes available, serious issues remain, including: design model and analysis model separation, geometry discretization, and material composition discretization and so on. In this paper, we propose Material-Based Graded Element which can improve the results of any heterogeneous objects.

3. Geometry Representation

To represent and manipulate heterogeneous objects within the computer and to develop the ability of describing how material composition varies within a heterogeneous objects. The reason for this is because the interior of heterogeneous objects is decomposed into a set of sub-regions. These sub- regions can be considered as the finite elements in this paper. It is also necessary to build up a data structure to represent the model which is created by FEM. This model can be considered as a 2D finite element model with material information, including some geometry elements like a node, edge, face and finite element, the topology relationship.

4. Material-based graded element formulation

There are two preconditions to design material distribution of Heterogeneous Objects. The first condition is the material composition of each node. The second condition is the material composition of discretional point inside each finite element can also be achieved if the first condition is satisfied. On the basis of these two conditions material property of each element is defined.

2D formulation

Material property of heterogeneous object is different in each point in the domain as shown in (Figure 1) and figure shows (Figure 2) that an element having different material property at nodes. For overcome with this problem assigning material property in D matrix (called material matrix) of each element in four different ways and considering only Young modulus only as a material property only as follows:

1. Average Young's modulus of domain
2. Average Young's modulus of element
3. Bi-Linear variation of Young's modulus along X & Y direction
4. Bi-Non linear variation of Young's modulus along X & Y direction

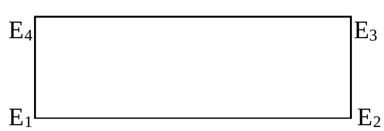


Figure 2: An element having different material property at nodes

Stiffness matrix for two-dimensional elements is given as

$$k = \int_{-l/2}^{l/2}\int_{-w/2}^{w/2} t \times [Bt][D][B]dxdy$$

Where

B is strain displacement matrix

$$B=\begin{bmatrix} \frac{\partial N1}{\partial x} & 0 & \frac{\partial N2}{\partial x} & 0 & \frac{\partial N3}{\partial x} & 0 & \frac{\partial N4}{\partial x} & 0 \\ 0 & \frac{\partial N1}{\partial y} & 0 & \frac{\partial N2}{\partial y} & 0 & \frac{\partial N3}{\partial y} & 0 & \frac{\partial N4}{\partial y} \\ \frac{\partial N1}{\partial y} & \frac{\partial N1}{\partial x} & \frac{\partial N2}{\partial y} & \frac{\partial N2}{\partial x} & \frac{\partial N3}{\partial y} & \frac{\partial N3}{\partial x} & \frac{\partial N4}{\partial y} & \frac{\partial N4}{\partial x} \end{bmatrix}$$

N1, N2, N3, N4 are shape functions

Shape function for 2 Dimensional for linear rectangular element in global coordinates are given as
N1= (1-X/L)*(1-Y/W), N2=(X*Y)*(1-Y/W)

N3=(X*Y)/ (L*W), N4=(Y/W)*(1-X/L)
B^t= B transpose
t=thickness and
D= Material matrix
The material matrix [D] for above mentioned condition are given as

1. Average young modulus of domain

$$D = \frac{E}{(1+\upsilon)(1-2\upsilon)}\begin{bmatrix} 1-\upsilon & \upsilon & 0 \\ \upsilon & 1-\upsilon & 0 \\ 0 & 0 & (1-2\upsilon)/2 \end{bmatrix}$$

υ Poison ratio
E Young's modulus

2. Average young modulus of element

$$D = \frac{(E_1+E_2)/2}{(1+\upsilon)(1-2\upsilon)}\begin{bmatrix} 1-\upsilon & \upsilon & 0 \\ \upsilon & 1-\upsilon & 0 \\ 0 & 0 & (1-2\upsilon)/2 \end{bmatrix}$$

3. Bi-linear variation in element

$$D = \frac{E(x)}{(1+v)(1-2v)}\begin{bmatrix} 1-v & v & 0 \\ v & 1-v & 0 \\ 0 & 0 & \frac{1-2v}{2} \end{bmatrix}$$

Where

$$E(x) = E_1 + (E_2 - E_1) \times \frac{x}{l} + (E_3 - E_1) \times \frac{y}{l} + (E_4 - E_3 - E_2 - E_1) \times (x \times y)/l^2$$

4. Bi- non-linear variation in element

$$D = \frac{E(x)}{(1+v)(1-2v)}\begin{bmatrix} 1-v & v & 0 \\ v & 1-v & 0 \\ 0 & 0 & (1-2v)/2 \end{bmatrix}$$

Where

$$E(x) = E_1 + (E_2 - E_1) \times \frac{x^2}{l^2} + (E_3 - E_1) \times \frac{y^2}{l^2} + (E_4 - E_3 - E_2 - E_1) \times (x^2 \times y^2)/l^4$$

Element properties are assembled to get global properties of structure by using following system equation

$$[K][\delta] = [F]$$

For 2Dimensional

A heterogeneous object with different material properties variation with two different materials is shown in Figure 3, 4, 5. The Young's modulus of material 1 is 1000 GPa and material 2 is 10 GPa, Poison ratio constant for both materials (υ) is 0.25 is considered. The dimensions of bar are 20x10x10 cm taken.

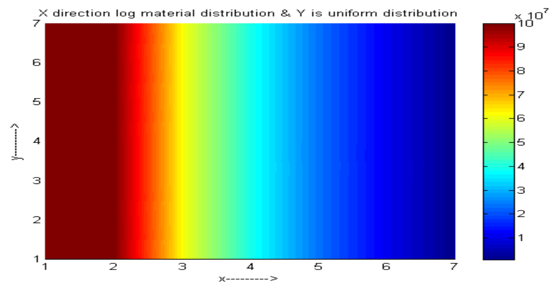


Figure 3: X direction material distribution log form & Y direction material distribution is uniform

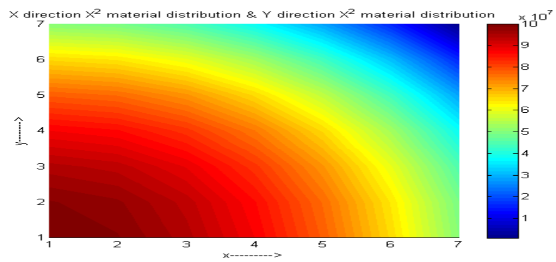


Figure4: X & Y direction material distribution is square form

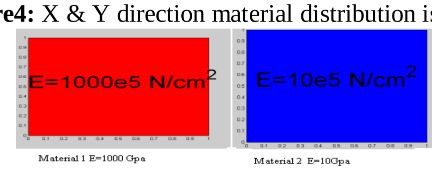


Figure 5: Model heterogeneous object

Loading Conditions

One end of the bar is kept to be fixed and a point load of 100 KN is applied in X direction as shown in (Figure 6, 7) as a *Load Case 1& Load Case 2.*

1) Point load at the corners
2) Point load at the centre

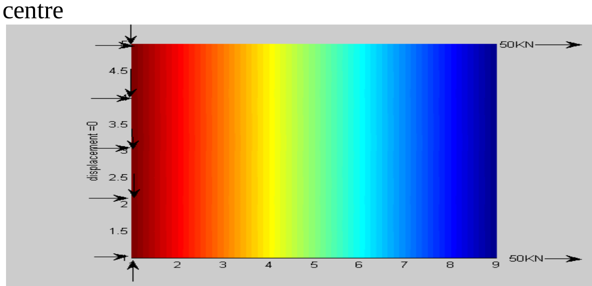


Figure 6: Point load at corners

Vol.4 Issue 5, May 2016 Pgs: 130-139

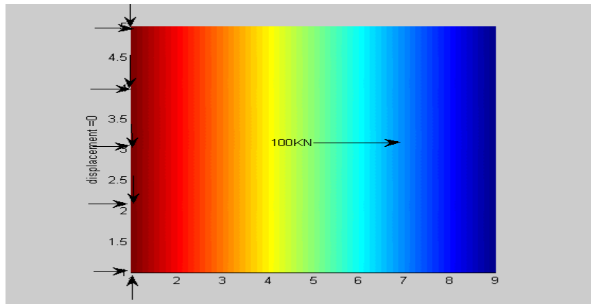


Figure 7: Point load at centre

Solution Techniques

A heterogeneous object (HO) is referred to as a solid component consisting of two or more material primitives distributed continuously or discontinuously within the object. It means each and every point material property material property, so for this type of material for getting exact result we try to define new type of element with the help of this we can come closer to the exact result.

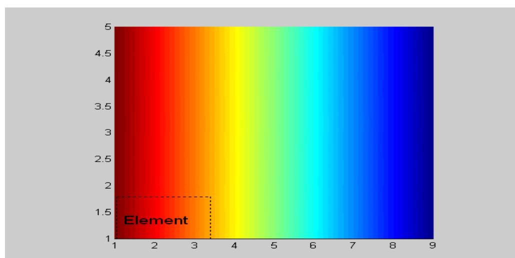
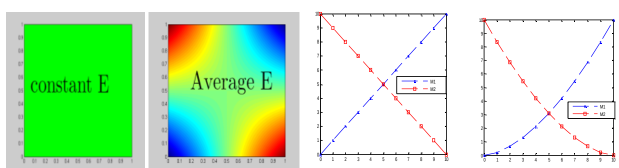


Figure 8: Domain with specified element



(a) (b) (c) (d)

Figure 14: Domain with specified element. (a) Average material property of domain (b) Average material property of element (c) Linear or Bi Linear variation of material property within element (d) Non linearly or bi non linear variation of material property within element

5. Results

The elastic analysis for two different type heterogeneous objects is submitted to static load as defined and the exact solution is obtained. In order to give the clear explanation for the behaviour of two heterogeneous objects, a homogeneous object is taken with average young's modulus of 505e5GPa with symmetry boundary and loading condition and symmetry result (i.e. both end is 0.0103cm displacement in X direction) are obtain as shown in (Load case 1) .The change in flow of displacement of two same heterogeneous object are obtained by using Material based graded and the number of elements is taken 7x7 so that the difference in result clears visible. The contour result shows the variation in the field variable (X displacement) in two heterogeneous objects.

Table 4: Load Case 1(load at edge)

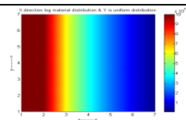
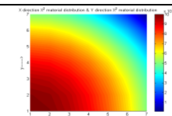
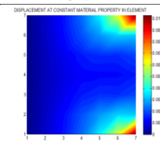
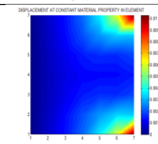
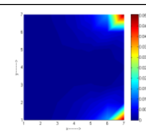
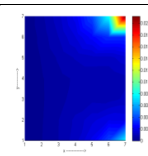
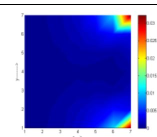
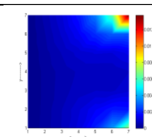
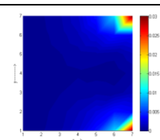
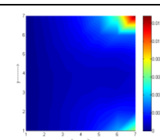
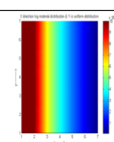
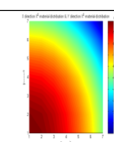
	Heterogeneous Objects	
Material Based Graded Element		
Average young modulus of domain		
Average Young's modulus of element		
Bi-Linear variation of Young's modulus along X & Y direction		
Bi-Non linear variation of Young's modulus along X & Y direction		

Table 5: Load Case 1(load at edge)

Material Based Graded Element		Heterogeneous Objects	
			
Average young modulus of domain	Maximum displacement at 1st load point (cm)	0.0103	0.0103
	Maximum displacement at 2st load point (cm)	0.0103	0.0103
Average Young's modulus of element	Maximum displacement at 1st load point (cm)	0.0505	0.0077
	Maximum displacement at 2st load point (cm)	0.0505	0.0212

Bi-Linear variation of Young's modulus along X & Y direction	Maximum displacement at 1st load point (cm)	0.0323	0.0076
	Maximum displacement at 2st load point (cm)	0.0323	0.0138
Bi-Non linear variation of Young's modulus along X & Y direction	Maximum displacement at 1st load point (cm)	0.0302	0.0069
	Maximum displacement at 2st load point (cm)	0.0302	0.0128

Table 6: Load Case 1(load at centre)

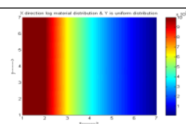
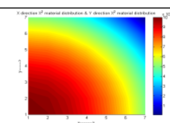
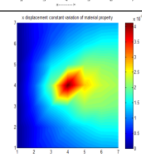
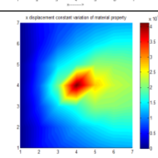
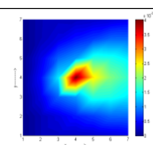
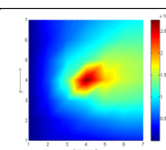
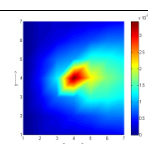
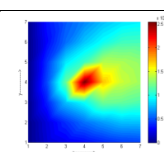
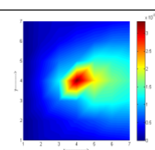
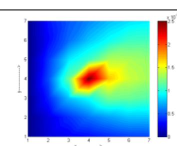
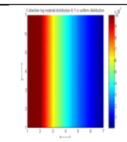
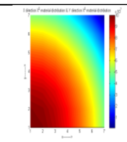
Material Based Graded Element	Heterogeneous Objects	
		
Average young modulus of domain		
Average Young's modulus of element		
Bi-Linear variation of Young's modulus along X & Y direction		
Bi-Non linear variation of Young's modulus along X & Y direction		

Table 7: Load Case 1(load at centre)

Material Based Graded Element		Heterogeneous Objects	
			
Average young modulus of domain	Maximum displacement at load point (cm)	0.0043	0.0043
Average Young's modulus of element	Maximum displacement at load point (cm)	0.004	0.0028
Bi-Linear variation of Young's modulus along X & Y direction	Maximum displacement at load point (cm))	0.0034	0.0025
Bi-Non linear variation of Young's modulus along X & Y direction	Maximum displacement at load point (cm)	0.0033	0.0025

6. Conclusion

The goal of the study was to introduce heterogeneous object with Material based graded element finite element analysis. Generally the accuracy prediction of field variables in heterogeneous object is depending upon accurate representation of material property and mesh sizes. In our approach material property is calculate in every point in the domain and increasing the number of points can increase the accuracy of domain and considering the variation with in the element by using this new type of element results improves which is seen load case 1 and case 2 for different material variation

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