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Finite Element Analysis of 1 D Heterogeneous Object Using Material Based Graded Elements

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ABSTRACT

A heterogeneous object is referred to a solid object made of different constituent materials. The object is made up of finite collection of regions having set of prescribed material classes of continuously or discontinuously varying material properties. These properties have an abrupt or gradual change across the interference of the material regions. Application of heterogeneous material range from bio-material products, aerospace structures, to meta-material with demonstrated performance improvement in structural, mechanical, electronics and or sensing property in these application , the performance goal are achieved because of the capability of varying material properties globally across the spatial domain. In order to take the full advantage of the greatest potential of heterogeneous object, we must go for developing their computer modelling and analysis techniques.

Based upon the computerized object model, finite element methods are often used to analyse these objects. In homogeneous objects, such analysis typically involves lengthy analysis model setup time, i.e. material property, mesh construction and mesh refinement, boundary condition setup and so on. It is estimated that analysis model setup takes more than 80% of the overall analysis time. In the context of heterogeneous object analysis, this issue is further exacerbated. In case of heterogeneous objects each and every point in the domain material property is vary (i.e. young modulus and Poisson ratio) so for this type of material for getting exact result we define new type of elements (called as Material Based Graded Element) in the domain with the help of this we can come closer to the exact result and also find the difference. The objective of the paper is to get near to accurate result of heterogeneous object by using Material-Based Graded elements.

In this paper we have considered linear and nonlinear material variation with two approaches average and integration approach for 1D and four types of graded element, Average Young's modulus of domain, Average Young's modulus of element.

***Keywords*:** Heterogeneous Objects, Material-Based Graded elements, Young's modulus, Linear, Non-linear.

1. INTRODUCTION

The term heterogeneous object refers to the non-uniformity of the material composition of objects as shown in (Figure 1) having two different material. No matter whether it is homogeneous or heterogeneous object its physical properties are influenced by its micro and macro structure significantly therefore a practical and realistic heterogeneous model is heterogeneous in both material composition and structure.

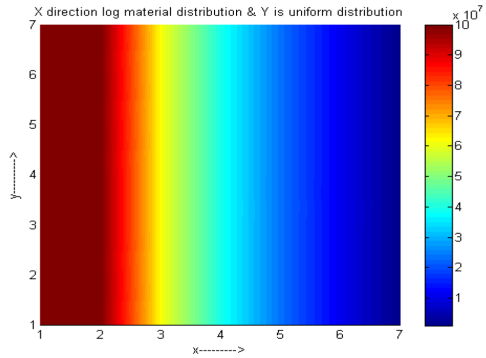

Figure 1: Model heterogeneous object

Heterogeneous objects are generally classified as objects with clear domain and those with continuous material variation. Current development of modelling system is concern with representing and controlling material composition within object boundary [8-9]. Heterogeneous objects have gained great popularities and research interest in the past few decades. Different from traditional parts which are made of the same uniform material (e.g. like copper cylinder or ceramics pistons).Heterogeneous objects refer to object with spatially different material composition or structures. Multi material objects embedded sensors /actuators, objects with functionally graded material (FGM) distributions, micro-electro-mechanical (MEMS) device, and porous structures and composite are typical examples of heterogeneous objects [10].

Based upon the computerized object model, finite element methods are often used to analyse these objects. In homogeneous objects, such analysis typically involves lengthy analysis model setup time, i.e. material property, mesh construction and mesh refinement, boundary condition setup and so on. It is estimated that analysis model setup takes more than 80% of the overall analysis time. In the context of heterogeneous object analysis, this issue is further exacerbated. In case of heterogeneous objects each and every point in the domain material property is vary (i.e. *Young modulus and Poisson ratio)* so for this type of material for getting exact result we define new type of elements (called as Material Based Graded Element) in the domain with the help of this we can come closer to the exact result and also find the difference.

2. Literature Review

Heterogeneous objects have been analysed by both theoretically and experimentally by many researchers and they have worked to optimize the various parameters such as material property, meshing etc.

In order to have some insight into present state of knowledge, extensive literature survey has been conducted. An attempt has also been made to highlight the gaps in the knowledge regarding to the material property assigning in the heterogeneous object and analysis of heterogeneous object which in turn has help to decide the scope of the present study. Two-dimensional computer simulation of deformation process in models of heterogeneous materials analysis and description of the behaviour of composite and porous materials under impact loading [1]. A numerical method reported for Analysis of two different kinds of heterogeneous elastic multi-layered cylinder with different value for both elastic modulus and poison ratio and another is an elastic hollow cylinder with continuously grade material properties [2]. The behaviour of many of the heterogeneous structural material is submitted like concrete, has different behaviour under tensile stresses in comparison to their behaviour under compressive stresses [3].

The aim of the author was to interpret behaviour of such materials subjected to tensile stress, by using newly introduce concept of fractal geometry. A Comparison is presented between subject-specific finite element analyses (FEA) of a human femur to experimental measurements, using different methods for assigning material property to the FE model [4]. An integrated design and analysis approach for heterogeneous object realization, which employs a unified design and analysis model based on B-spline representation and allows for direct interaction between the design and analysis model without laborious meshing operation. In the design module, a new approach for intuitively modeling of multi-material objects, termed heterogeneous lofting, is presented [7].

Two specific patient finite element model were create and analysis under the compressive load. Vertebral bone was treated as an orthotropic material. Commercial software for medical image processing (CT imaging) and FE analysis ANSIS were used for mechanical analysis of the FE model [3]. B-spline surface represent material composition solid model of proximal femur graded element is used to defining inhomogeneous isotropic elastic property in the finite element model to improve the performance. Although there are many heterogeneous object modelling and analysis schemes available, serious issues remain, including: design model and analysis model separation, geometry discretization, and material composition discretization and so on. In this paper, we propose Material-Based Graded Element which can improve the results of any heterogeneous objects.

3. Geometry Representation

To represent and manipulate heterogeneous objects within the computer and to develop the ability of describing how material composition varies within a heterogeneous objects. The reason for this is because the interior of heterogeneous objects is decomposed into a set of sub-regions. These sub- regions can be considered as the finite elements in this paper. It is also necessary to build up a data structure to represent the model which is created by FEM. This model can be considered as a 2D finite element model with material information, including some geometry elements like a node, edge, face and finite element, the topology relationship.

4. Material-based graded element formulation

There are two preconditions to design material distribution of Heterogeneous Objects. The first condition is the material composition of each node. The second condition is the material composition of discretional point inside each finite element can also be achieved if the first condition is satisfied. On the basis of these two conditions material property of each element is defined.

1D formulation

Finite element analysis is a numerical technique in this method all the complexities of the problem like varying shape, boundary condition and loads are maintained but the solution obtained are approximate because of its diversity and flexibility. In engineering problem there are some basic unknown / field variable if they are found, the behaviour of structure predicted like displacement in solid mechanics. For 1 D only U is considered i.e. displacement in X direction only, and it is given by

$$U=\sum_{i=1}^{n} N_i \quad U_i$$

Where N is shape function, for 1D it is given as,

N1= (X2 - X)/L, N2 = (X-X1)/L

X2, X1 is end points of the element, L length of the element; X is any point in domain.

Stiffness matrix is given as

$$k = \int_{x_1}^{x_2} [B^t][d][B] \partial x$$

Where B is strain displacement matrix

B=1/L*[-1 1]
B^t is B transpose
D is material matrix.

In 1D analysis we have considered variation of material in terms of variation of Young's modulus only and used two approaches to take account of material variation.

Average Approach:-

In this case we have calculated the value of Young's modulus at each node of element by using equation of material variation and elemental Young's modulus is taken as average of nodal Young's modulus.

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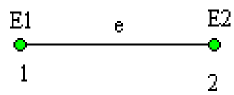


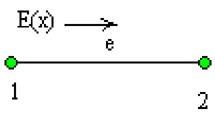
$$D = E1 + E2 / 2$$

Therefore material stiffness matrix becomes

$$Ke = \frac{((E1+E2)/2)*Ae}{Le} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Integration Approach:-

In this case we have calculated the value of Young's modulus element by directly integrating equation of material variation.



$$D = \int_{X1}^{X2} E(x)dx$$

Therefore material stiffness matrix becomes,

$$Ke = \frac{Ae}{Le^2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \int_{X1}^{X2} E(x)dx$$

5. Numerical Results and Discussion

Simple bar of length 5m, Area of c/s 1 cm^2 is considerd, Tensile load of 4000N is applied at one end and other end is fixed, material variation is $E(x)=E_0(1-0.5(x/L)$^n), & E_0=200Gpa taken.

Results obtain are tabulated as, Table 1 shows values of maximum displacement with different no. of elements for both average and integration approach with linear material variation. It is clear that as number of element increases diplacement increases. Also it is clear that values of displacements is same for both cases. This is because in linear variation integration over elemental lenght leads to averaging. So there is no any truncation error.

Also Figure 2 Showes that in case of integration approach we get convergence with less number of elements than in average case.

Table.1: Displacement variation for linear case

Number of Element	Maximum Displacement (mm)	
	Average Approach	Integration Approach
5	0.97842246479	0.97842246479
10	0.97933762001	0.97933762001
15	0.97950796577	0.97950796577
20	0.97956765206	0.97956765206
25	0.97959528978	0.97959528978
30	0.97961030592	0.97961030592
40	0.97962523887	0.97962523887
50	0.97963215141	0.97963215141
100	0.97964136884	0.97964136884
250	0.97964394987	0.97964394987

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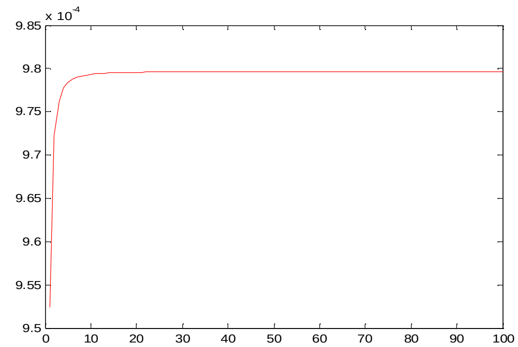

Figure 2: Convergence for Avarage pproach (n=1)

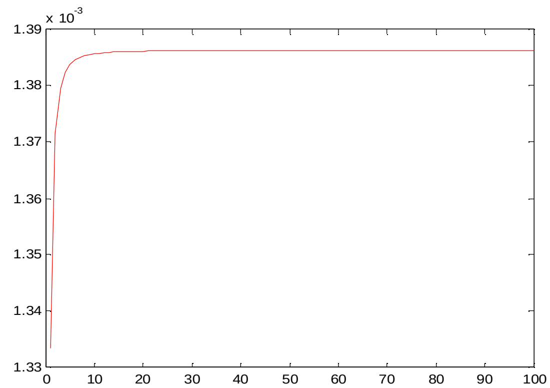

Figure 3: Convergence for Integration pproach (n=1)

Table 2 showes values of maximum displacement with different no. of elements for both average and integration approach with non-linear material variation(n=2). It is clear that as number of element increases diplacement increases in both cases. It is that in case of integration approach we get convergence with less number of elements than in average case.

Table.2: Displacement variation for Non-linear case (n=2)

Number of Element	Maximum Displacement (mm)	
	Average Approach	Integration Approach
5	1.081638110014	1.086476168023
10	1.086262144202	1.087478787378
15	1.087123516269	1.087664837005
20	1.087425373336	1.087729982647
25	1.087565156289	1.087760140746
30	1.087641105354	1.087776524243
40	1.087716635168	1.087792815575
50	1.087751598827	1.087800356445
100	1.087798221122	1.087810411245
250	1.087811276201	1.087813226652

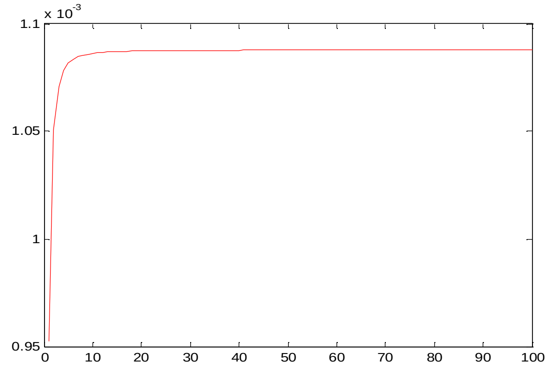

Figure 4: Convergence for Avarage pproach (n=2)

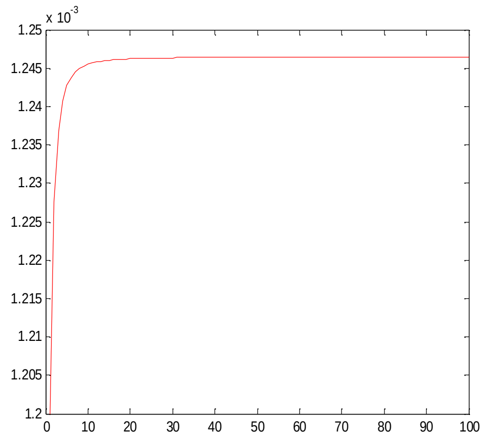

Figure 5: Convergence for Integration pproach (n=2)

Table 3 showes maximum displacement for non-linear material variation(n=3).As non-linearity increses displacement increases.

Table.3: Displacement variation for Non-linear case (n=3)

Number of Element	Maximum Displacement (mm)	
	Average Approach	Integration Approach
5	1.137520258669	1.144209278180
10	1.143815216654	1.145495558343
15	1.144986755492	1.145734254359
20	1.145397233159	1.145817835835
25	1.145587303169	1.145856528813
30	1.145690571673	1.145877549002
40	1.145793267818	1.145898451003
50	1.145840806325	1.145908126064
100	1.145904195791	1.145921026560
250	1.145921945824	1.145924638783

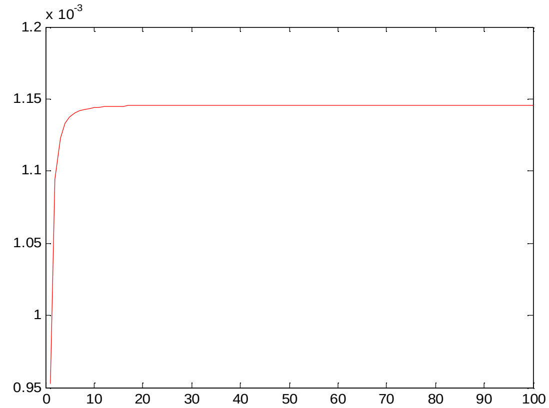

Figure 6: Convergence for Avarage pproach (n=3)

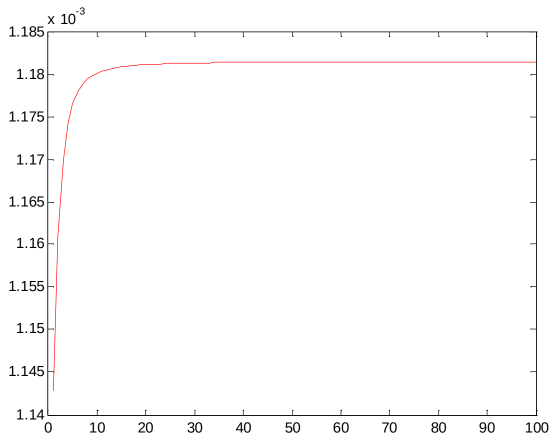

Figure 7: Convergence for Integration pproach (n=3)

6. Conclusion

The goal of the study was to introduce heterogeneous object with Material based graded element finite element analysis. Generally the accuracy prediction of field variables in heterogeneous object is depending upon accurate representation of material property and mesh sizes. In our approach material property is calculate in every point in the domain and increasing the number of points can increase the accuracy of domain and considering the variation within the element by using this new type of element results improves which is seen for different material variation

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