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Effect of selectable parameters on performance of Meshless Local Petrov Galerkin (MLPG) method

Jay N. Vyas[1], Sachin D. Daxini[2], Anjali N. Dave[3], J. M. Prajapati[4]

[1]B.H.Gardi College of Engg. & Tech., jayvyas36@gamil.com
[2]Babaria Institute of Technology, sachin_daxini@rediffmail.com
[3]B.H.Gardi College of Engg. & Tech., andave@gardividyapith.ac.in
[4]M. S. University, Vadodara, drjmprajapati@gmail.com
Author Correspondence: Mechanical Department, "Gardi Vidyapith" Rajkot- Kalawad Highway, Village: Anandpar Rajkot - 361 162 Gujarat , India.9016378856, jayvyas36@gmail.com

Abstract

Meshless methods (MM) are alternative numerical techniques developed to eliminate some evident issues of conventional mesh based methods like Finite Element Method (FEM) in dealing with problems of crack propagation, large deformation and simulation of some manufacturing process. FEM also suffers from discontinuous secondary field variable like stress across element boundary. Meshless Local Petrov Galerkin (MLPG) method is a meshfree technique developed and used in various fields of engineering till now. The method is based on a local weak form of the governing differential equation and uses Moving Least Square (MLS) approximants as shape functions. While formulating problem in MLPG method, selection of weight function, number of nodes in the problem domain and size of support domain and subdomain plays an important role and affects solution accuracy. Present work shows MLPG formulation for one dimensional bar problems with body force and a systematic approach for evaluating the performance of MLPG method when different selectable parameters are incorporated in the solution like different weight functions, different node schemes in the problem domain and different size of support domain and subdomain. Results obtained with MLPG method are compared with exact analytical solution and standard FEM solution. The content of the paper will be helpful to the beginner in the area of meshfree method.

Keywords: Meshless Method, Meshless Local Petrov Galerkin method, weight function, number of nodes, size of support domain and subdomain.

1. Introduction

Finite Element Method (FEM) is an established numerical solution technique for engineering problems in various fields. But it is not suitable to special class of problems like moving material discontinuity problems, large deformation problems and simulation of some manufacturing process like extrusion. FEM also suffers from the issue of discontinuous secondary variables like stresses across element boundaries. MMs were developed as an alternative numerical approach to eliminate these drawbacks. The main objective of these methods is to reduce the difficulty of meshing and remeshing of the complex structural problem domains. Descritization procedure of the problem domain is done only by simply adding or deleting nodes where desired. Predefined nodal connectivity is also not needed, only nodal coordinates and their domain of influence (DOI) are necessary [1]. There are several MMs developed till now which includes, Smooth Particle Hydrodynamics (SPH) method proposed by Gingold and Monaghan [2], Element-Free Galerkin (EFG) method proposed by Belytschko et al. [3], the Reproducing Kernel Particle Method (RKPM) proposed by Liu et al. [4], Meshless Local Petrov-Galerkin (MLPG) method proposed by Atluri et al. [8], and some other methods proposed later. The objective of present work is to study and check performance of MLPG method. The method is based on a local weak form of the governing differential equation and uses shape functions derived from moving least-square (MLS) approximation. The method is flexible in choice of trial and test functions and accordingly Atluri and Shen have derived six MLPG formulations depending on various test functions applied and marked them MLPG 1– MLPG 6. MLPG has been successfully applied to various problems in different areas of engineering. Review paper on MLPG application in engineering and science was presented by J. Sladek, P. Stanak, Z. D. Han, V. Sladek and S.N. Atluri [5], and in structure and fracture mechanics was presented by S. Daxini and J.M. Prajapati [6] recently.

Present work evaluates the performance of MLPG method for different weight functions, different node schemes in problem domain and different size of support domain and subdomain for one dimensional bar problem. Results obtained with MLPG method are compared with exact solution and standard FEM solution to show relative significance of these parameters in MLPG formulation. The discussion begins with problem definition and MLPG formulation for bar with body force in Section 2. Performance evaluation of MLPG method for various parameters is discussed in Section 3 and Section 4 presents conclusion of the work.

2. Problem definition and MLPG formulation

One dimensional structural problem of a bar fixed at one end and subjected to linear body force is shown in the Figure 1 with unit length and unit cross sectional area.

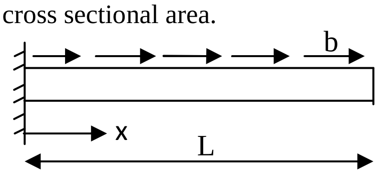

Figure 1 Axial deformation of a bar under linear body force

Weak form for 1-D bar problems is given as [8]:

$$-\frac{d}{dx}\left(b(x)\frac{du}{dx}\right)+c(x)u = f(x) \tag{2.1}$$

in domain Ω (0 ≤ x ≤ L) with boundary Γ, where b(x) and c(x) are problem parameters that may be functions of the coordinate x, and f(x) is any load which may also be a function of x. The essential and natural boundary conditions (NBC) are of the form,

$$u = \bar{u} \text{ on } \Gamma_u, \; q = \bar{q} \text{ on } \Gamma_q \tag{2.2}$$

Where, q = b(du/dx), Γ_u and Γ_q denote the boundary region where primary variable, u and secondary variable, q are prescribed, respectively. In 1-D problems, these boundary regions are the points x=0 and x=L. For the problem of axial deformation of a bar, the primary variable u is longitudinal displacement, b = EA where, E is elastic modulus and A is cross sectional area, f is the applied body force and b (du/dx), the secondary variable is the axial force. Exact analytical solution for the above problem is given by [3]:

$$u_x = \frac{1}{E}\left(\frac{x}{2} - \frac{x^3}{6}\right) \tag{2.3}$$

2.1 MLPG formulation

Node generation is the first step in any meshfree formulation. Generation of shape functions through nodes is the next procedural step followed by generation of discrete equations based on global or local weak form and cell based integration. Global equations are generated by assembling discrete equations and EBCs are imposed by appropriate techniques like Penalty approach, Lagrange multiplier, and coupling with FEM etc. Solution is obtained for displacements and stresses can be derived from it afterwards [1]. In present problem, bar geometry is defined with eleven nodes as follows:

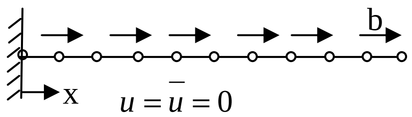

Figure 2 Node generation in a bar

A local weak form is developed from the classical weighted-residual form of the governing differential equation. MLS interpolation is used to construct the approximations to the solution known as trial functions or shape functions. Test functions are chosen from a different space than the trial functions and Essential Boundary Conditions (EBC) are enforced by a penalty method similar to FEM. A system of algebraic equations is derived by substituting the trial (shape) and test functions into the local weak form. To obtain an approximate solution of eq. (2.1), weighted residual technique is employed. For approximate solution, the residual or error term is given by [7]:

$$R = -\frac{d}{dx}\left(b(x)\frac{du}{dx}\right) + c(x)u - f \tag{2.4}$$

Control of the errors is affected by multiplying residual by a weight function v(x) and integrating over the whole domain, and finally setting the integral to zero [8].

$$\int_\Omega v\left[-\frac{d}{dx}\left(b\frac{du}{dx}\right) + cu - f\right]dx = 0 \tag{2.5}$$

Eq. (2.5) shows the classical weighted residual form of the governing differential equation. The weak form of the weighted residual equation is obtained by transferring the differentiation from the primary variable u to the weight function v. This is achieved by integrating by parts in 1-D problems and integrating by parts gives,

$$\int_\Omega b\frac{du}{dx}\frac{dv}{dx}dx + \int_\Omega cvudx - \int_\Omega fvdx - \left[vb\frac{du}{dx}\right]_\Gamma = 0 \tag{2.6}$$

Unlike conventional method, which selects v=δu i.e. the test function is chosen as variation of u, Atluri and Zhu proposed selection of weight function from different space. Test functions with well defined shapes and whose values vanish at a certain controllable distance are selected. Based on these functions, integration can be restricted to local sub-domains. The weak form is therefore written for the local sub-domain Ω_s as,

$$\int_{\Omega s} b\frac{du}{dx}\frac{dv}{dx}dx + \int_{\Omega s} cvudx - \int_{\Omega s} fvdx - \left[vb\frac{du}{dx}\right]_{\Gamma s} = 0 \qquad (2.7)$$

With penalty parameter for enforcing EBCs and considering possibility of intersection of local boundary with global boundary, local weak form of the above equation is written as [8],

$$\int_{\Omega s} b\frac{du}{dx}\frac{dv}{dx}dx + \int_{\Omega s} cvudx - \int_{\Omega s} fvdx + \alpha_u\left[(u-\bar{u})v\right]_{\Gamma su} - [vq]_{\Gamma su} - [v\bar{q}]_{\Gamma sq} = 0 \qquad (2.8)$$

In above expression, the primary variable u and its derivative is substituted by,

$$u^h(x) = \sum_{j=1}^{n}\phi_j(x)\hat{u}_j, \quad \frac{du^h}{dx} = \sum_{j=1}^{n}\frac{d\phi_j}{dx}\hat{u}_j$$

where, ϕ_j are shape functions and $\hat{u}_j$ are factious nodal values of u. The MLS interpolations are used to form the shape (trial) functions, *u*, in the implementation of the MLPG method. The MLS approximant $u^h(x)$ of the function u(x) defined over the domain Ω, is given by [7,8]

$$u^h(x) = p^T(x)a(x), \forall x \in \Omega \qquad (2.8)$$

where, $p^T(x)$ are monomial basis functions of order m and a(x) are vector coefficients which are functions of space coordinates x, which can be determined at any point x by minimizing weighted discrete L_2 norm defined as follows [7,8],

$$J(x) = \sum_{i=1}^{n} w_i(x-x_i)\left[p^T(x_i)a(x)-u_i\right]^2 \qquad (2.9)$$

While the test functions are chosen as weight functions in eq. (2.8). The choice of the weight function affects the resulting approximation in meshfree solution and it should be constructed in such a way that their value should decrease as the distance from x to x_i increases. Some frequently used weight functions are given below:

Cubic spline weight function,

$$\begin{aligned} w_i(x) &= \frac{2}{3} - 4r^2 + 4r^3, \qquad 0 \le r \le \frac{1}{2} \\ &= \frac{4}{3} - 4r + 4r^2 - \frac{4}{3}r^3, \qquad \frac{1}{2} \le r \le 1 \\ &= 0, \quad r > 1 \end{aligned} \qquad (2.10)$$

Quartic spline weight function,

$$\begin{aligned} w_i(x) &= 1 - 6r^2 + 8r^3 - 3r^4, \qquad 0 \le r \le 1 \\ &= 0, \quad r > 1 \end{aligned} \qquad (2.11)$$

Compact Support Radial Basis Function (CSRBF),

$$\begin{aligned} w_i(x) &= (1-r)^6[6 + 36r + 82r^2 + 72r^3 + 30r^4 + 5r^5], \qquad 0 \le r \le 1 \\ &= 0, \quad r > 1 \end{aligned} \qquad (2.12)$$

In the above expressions $r = d_i/R_i$, where $d_i = |x-x_i|$ is the distance between node x_i and x while R_i is the size of the support for weight function w_i and determines support of node x_i. The size of support, R_i, of the weight function w_i associated with node i should be chosen such that R_i should be large enough to have sufficient number of nodes covered in the domain of definition of every sample point to ensure the regularity of matrices [6]. On substitution of trial and test functions into eq. (2.8), resulting system of equations [8],

$$K^{(node)}u + K^{(bdry)}u - f^{(node)} - f^{(bdry)} = 0 \qquad (2.13)$$

where,

$$K_{ij}^{node} = \int_{\Omega^i s} b \frac{dv_i}{dx}\frac{d\phi_j}{dx}dx + \int_{\Omega^i s} cv_i\phi_j dx, \qquad K_{ij}^{bdry} = \left[\alpha v_i \phi_j\right]_{\Gamma^i su} - \left[v_i \frac{d\phi_j}{dx}\right]_{\Gamma^i su}$$

$$f_i^{node} = \int_{\Omega^i s} fv_i dx, \qquad f_i^{bdry} = \left[\alpha \bar{u} v_i\right]_{\Gamma^i su} + \left[v\bar{q}\right]_{\Gamma^i sq}$$

The stiffness matrix of the system will be banded but unsymmetric. But MLPG method results in truly meshless method which does not require elements either for interpolation or for integration purpose. Systematic procedural steps for MLPG implementation were given by Atluri and Zhu [6]. In subsequent sections, effect of weight function selection, number of nodes in problem domain and size of support domain and subdomain is checked for their significance in MLPG formulation.

3. Effect of various parameters on MLPG performance

The bar problem with body force, as defined and formulated above, is studied in detail with the help of MATLAB program. The performance of MLPG method is checked with different weight functions, number of nodes in the domain and size of support domain and subdomain in this section.

3.1 Selection of the Weight function

As mentioned earlier, choice of weight function plays an important role in meshfree solution. Most of the weight functions are bell shaped and some of the frequently used functions are: Cubic spline, quartic spline, CSRBF, exponential etc. as given in eq. (2.10) - (2.12). For one dimensional bar problem discussed above, all these weight functions are implemented in the solution and the results for displacements (d) are obtained and compared with exact solution obtained by eq. (2.3). They are tabulated and plotted below:

Table 1 Comparison of different weight functions for displacement

X	Exact Analytical solution (d)	MLPG, CSRBF (d)	MLPG, Quartic Spline (d)	MLPG, Cubic Spline (d)
0	0	0	0	0
0.1	0.0498	0.0498	0.0501	0.0638
0.2	0.0987	0.0985	0.0991	0.1057
0.3	0.1455	0.1452	0.1461	0.1475
0.4	0.1893	0.189	0.19	0.1874
0.5	0.2292	0.2287	0.2296	0.2233
0.6	0.264	0.2635	0.264	0.2548
0.7	0.2928	0.2923	0.292	0.2809
0.8	0.3147	0.314	0.3129	0.3007
0.9	0.3285	0.3278	0.3258	0.3132
1	0.3333	0.3326	0.3301	0.3178

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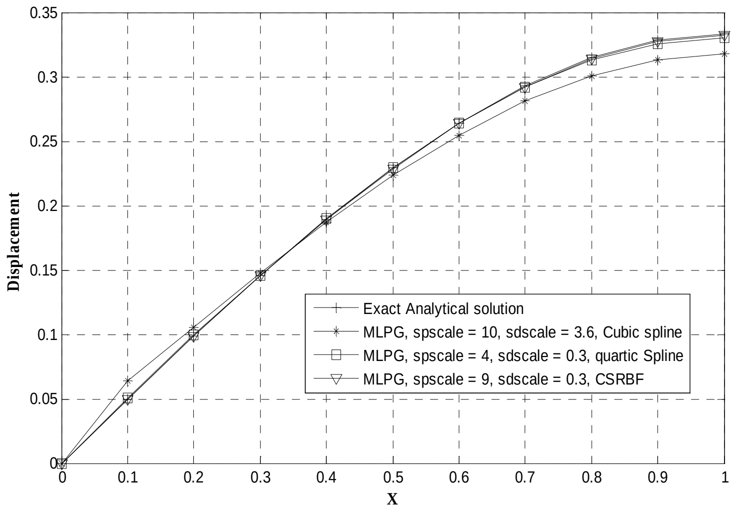

Figure 3 Comparison of different weight functions for displacement

Table 1 gives comparison of results for different weight functions while Figure 3 provides graphical representation of this exercise. It can be observed that CSRBF and Quartic Spline weight function are giving better results than Cubic Spline.

3.2 Different node schemes in problem domain

The problem domain of one dimensional bar, which was defined with eleven nodes initially, is now divided in six and twenty one nodes. For all node schemes, scale factor of sub domain is 0.3 and weight function is Quartic Spline. The scale factor of support domain is 5 for 21 nodes, 4 for 11 nodes and 6 for 6 nodes. Results of displacements (d) are obtained and compared with exact solution.

Table 2 Comparison of results for - 21 Nodes, 11 Nodes and 6 Nodes for displacement

X	**Exact analytical solution (d)**	**MLPG, 21 Nodes, Quartic Spline (d)**	**MLPG, 11 Nodes, Quartic Spline (d)**	**MLPG, 6 Nodes, Quartic Spline (d)**
0	0	0	0	0
0.05	0.025	0.0251		
0.1	0.0498	0.0501	0.0501	
0.15	0.0744	0.0748		
0.2	0.0987	0.0991	0.0991	0.0965
0.25	0.1224	0.1229		
0.3	0.1455	0.1461	0.1461	
0.35	0.1679	0.1685		
0.4	0.1893	0.19	0.19	0.1844
0.45	0.2098	0.2105		
0.5	0.2292	0.2298	0.2296	
0.55	0.2473	0.2478		
0.6	0.264	0.2644	0.264	0.2566
0.65	0.2792	0.2794		
0.7	0.2928	0.2927	0.292	
0.75	0.3047	0.3043		
0.8	0.3147	0.3138	0.3129	0.3055
0.85	0.3226	0.3214		
0.9	0.3285	0.3268	0.3258	
0.95	0.3321	0.33		
1	0.3333	0.3309	0.3301	0.3221

Comparison of results shows better approximation of results with higher number on nodes in the problem domain i.e results with 21 nodes are quite accurate than other two node schemes. Different values of scale factor for support domain and sub domain also affect the solution and hence their values should be selected appropriately.

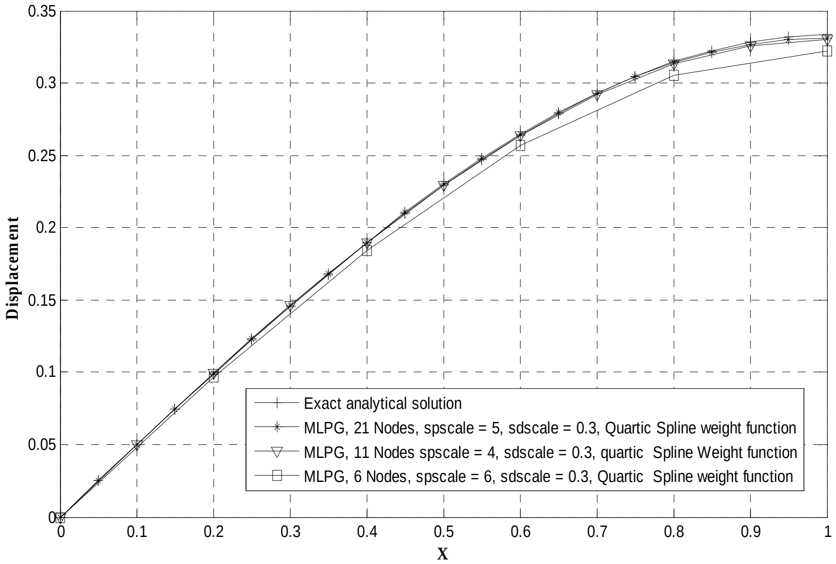

Figure 4 Comparison of results for- 21 Nodes, 11 Nodes and 6 Nodes for displacement

3.3 Comparison of results with FEM

FEM is applied to the bar problem now. The results for displacements are obtained and compared with MLPG solution. The problem domain is divided in ten simple line elements for FEM [8]. The results are tabulated and plotted below, which shows good agreement between these results.

Table 3 Comparison of results: MLPG and FEM for displacement

x	Exact Analytical solution (d)	MLPG, 11 Nodes, CSRBF (d)	FEM (d)
0	0	0	0
0.1	0.0498	0.0498	0.0497
0.2	0.0987	0.0985	0.0985
0.3	0.1455	0.1452	0.1452
0.4	0.1893	0.189	0.189
0.5	0.2292	0.2287	0.2288
0.6	0.264	0.2635	0.2636
0.7	0.2928	0.2923	0.2924
0.8	0.3147	0.314	0.3142
0.9	0.3285	0.3278	0.328
1	0.3333	0.3326	0.3328

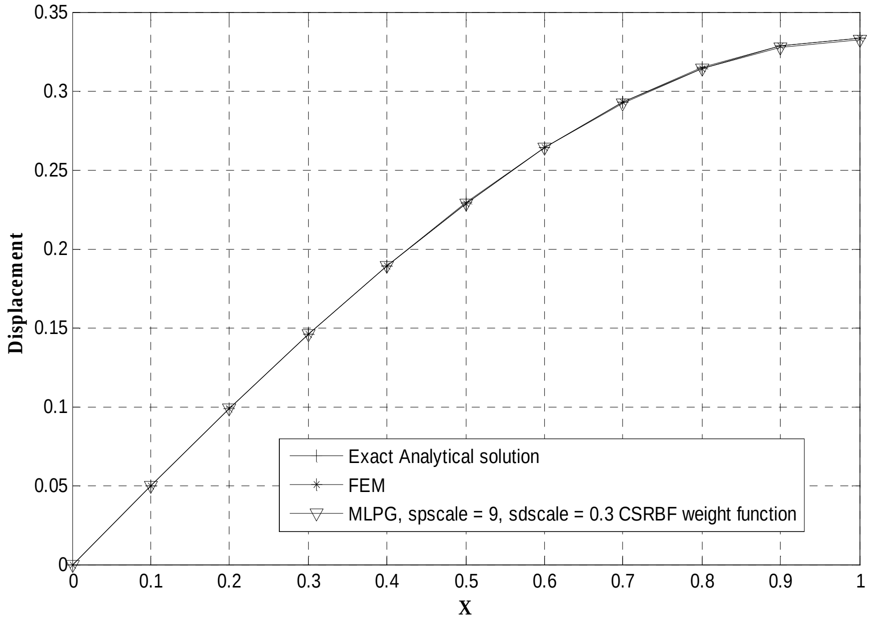

Figure 5 Comparison of results: MLPG and FEM for displacement

3.4 Comparison of stress results with FEM

One of the apparent drawback of FEM is discontious secondary varialbe like stress across element boundary. The same is realized when stress results of FEM are compared with MLPG stress results in Fig. 6. It can be observed that the MLPG stress results are contiouns and in good agreement with analytical solution while FEM stress results are discontious at element boundaries. Thus, meshless methods have proved to be more accurate than conventional mesh based methods.

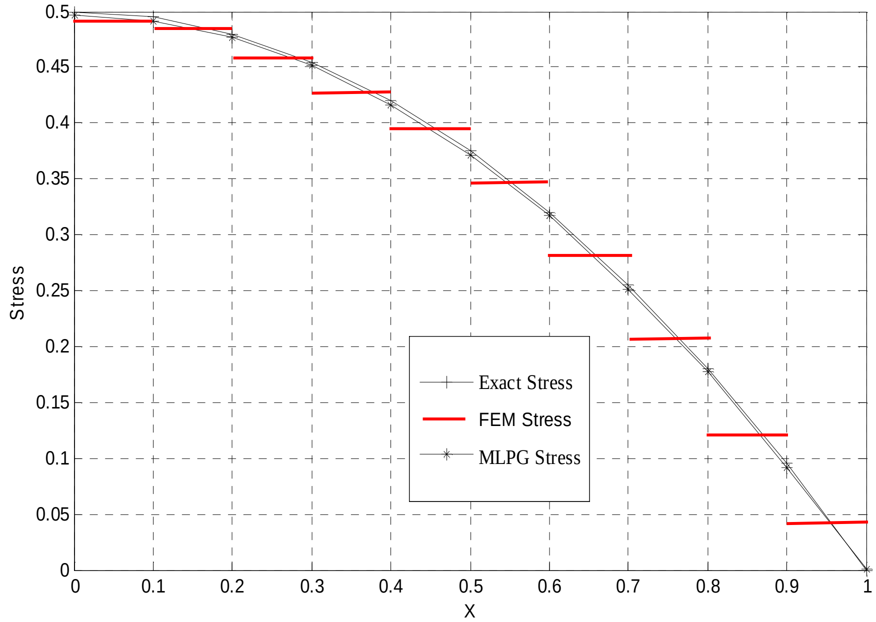

Fig. 6 Comparison of stress for FEM and MLPG

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4 Conclusion

MLPG method is a promising meshfree approach developed in recent time and successfully applied to various types of engineering applications. Objective of preset work is to check accuracy of MLPG solution for variation in different selectable parameters like weight functions, different node distribution schemes in the problem domain and different scale factor for support & sub domain. Well defined bar problem with linear body force is studied in detail. The results of MLPG were found in good agreement to FEM and exact solution. Convergence in meshless method is exemplary but for given accuracy, MLPG method is computationally more expensive than FEM. All the derived results for displacement are presented in tabular format and graphical form for direct/visual comparison. Higher number node scheme, CSRBF and Quartic Spline weight functions and appropriate values of scale factors in MLPG yield accurate results. The study carried out will be useful to any beginner or student in the area of meshfree method.

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Biography

Jay N. Vyas : He is a Student of M.E (CAD/CAM) in B.H.Gardi college of Engg. and Tech., Rajkot, Gujarat, India. He has interested to work in a new area as a Reasearch in that field. This paper is a part of that Technical work.

Mr. S. D. Daxini: He is presently working with Babaria Institute of Technology, Varnama, Vadoara, Gujarat, India as an Associate Professor in Mechanical Engineering Department. He completed his graduation in Mechanical Engineering from B.V.M. Engineering College, Vallabh Vidyanagar, Gujarat in 2001 and masters in CAD/CAM from L.D. College of Engineering, Ahmedabad, Gujarat in 2003. He has published several research papers in national and international journals and conferences. He has more than ten years of total experience including professional and academics.

Ms. A. N. Dave: She is presently working with B. H. Gardi college of Engg. and Technology, Rajkot, Gujarat, India as an Assistant Professor in Mechanical Engineering Department. She completed his graduation in Mechanical Engineering and masters in CAD/CAM.

Dr. J. M. Prajapati is presently working with M. S. University, Vadodara as a Reader in Mechanical Engineering Department. He completed his Ph. D. from Hemchandracharya North Gujarat University, Patan in 2009 and M.E. in Machine Design from S. P. University in 2003. He has published more than thirty research papers in national and international journals and conferences. His areas of interest are: Machine Design, Finite Element Methods in Design and Robotics. He has more than twenty years of total experience including professional and academics.