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**INTERNATIONAL JOURNAL OF RESEARCH IN  
AERONAUTICAL AND MECHANICAL ENGINEERING****DYNAMIC ANALYSIS OF NATURAL FREQUENCY AND MODE  
SHAPE OF CANTILEVER AND FREE-FREE CONDITION PLATE BY  
CONSIDERING UNCERTAIN PARAMETERS**Imran Ahemad Khan<sup>1</sup>, G. K. Awari<sup>2</sup>

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**Abstract**

Dynamic analysis of plates has been an important research subject and many technical researches are going on. The plate is made-up of a material may not be perfect it will contain some uncertain parameters. The concept of uncertainty plays an important role in the design of practical mechanical system. So it becomes important to study its effects on mechanical system for different frequency domain. Here in this paper structural element square plate with cantilever and free-free boundary condition is selected on which mass, stiffness and combination of it, uncertainty is taking in account. By using Finite Element Method technique finding out, how plate is dynamically behaves in vibration. During modal analysis natural frequency and mode shapes are finding and compared the response of bare plate and the plate with mass and mass-stiffness combination uncertainty.

**Keywords:** Mode Shape; Natural Frequency; Ansys Software; Uncertain Parameter; Modal Analysis.

**1. INTRODUCTION**

In engineering structures plates and plate-like components are widely used such as several aero space modal, machine structures, civil engineering structure, boards in electronic equipment. Vibration analysis of plates and plate-like components has been an important research subject. Predicting the dynamic response of a vibrating system generally involves determining the equations of motion of the structure and solving them in order to find the natural frequencies and mode shapes of the system for given boundary conditions (Jimin He et al.,2001). For more complex systems, the equations of motion can be approximated using various deterministic modelling techniques such as finite element analysis (FEA) (A.L. Morales et al., 2012) and dynamic stiffness techniques. In engineering design, it is important to calculate the response quantities such as the displacement, stress, vibration frequencies, and mode shapes of given set of design parameters (Xiao-Ming et al., 2008) Energy methods such as Statistical Energy Analysis (SEA) are appropriate dynamic predictive techniques at high frequencies. SEA is a technique which models a structure in terms of its gross system properties. In this method, a complete system is described in terms of energy flow between various subsystems (Cicirello et al.,

2010). The natural frequencies and mode shapes can then be used to predict the response due to an applied excitation (Sondipon et al., 2007). The study of mathematical models which involve physical and geometric parameters such as mass density  $\rho$ , elastic modulus  $E$ , Poisson's ratio  $\nu$ , lengths, and cross-section shape characteristics. In many practical engineering applications, these parameters frequently do not have well-defined values due to non-homogeneity of the mass distribution geometric properties or physical errors, as well as variation arising from the assembly and manufacturing processes, structural loading and environmental conditions (G. Manson, 2005). These methods are extensively used to predict the linear dynamic response of structures in the low frequency region. Finite element methods are one of the most widely used deterministic techniques. For this technique, a structure is divided into a number of elements. In engineering design these uncertainties in material properties, geometric parameters and boundary conditions are often unavoidable and must be considered. This concept of uncertainty plays an important role in investigation of various engineering problems (Imran Ahemad Khan et al., 2013). The dynamic behaviour of each element is derived and the solution of the structure as a whole is reconstituted by attaching each element together at its node points.

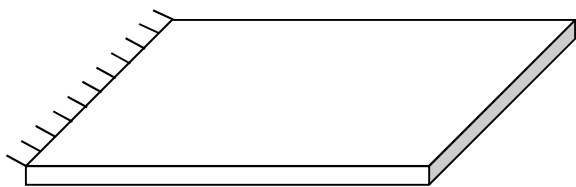
Here in this paper square plate is used with cantilever and free-free boundary condition. In this square plate two major uncertain parameters are consider i.e. mass and stiffness. Hence the analysis is done on bare plate and plate with uncertainty due to mass and combination of mass and stiffness. For all these combination modal analysis is done and different natural frequency and mode shapes were found.

## 2. Mathematical Modelling

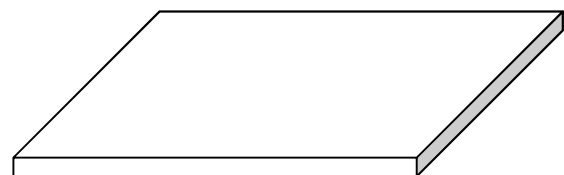
Using the Lagrange- Rayleigh -Ritz technique (Meirovitch L, 1986), the equations of motion of a dynamic plate in modal space can be derived. Initially bare plate dynamic is derived and then adding different uncertainty in the plate is derived.

### 2.1 Bare Rectangular Plate

Considering the cantilever and free- free bare square plate as shown in Figure 1 and Figure 2 (with no structural uncertainty). For these plates eigenfunction is described by sinusoidal mode shapes in the  $x$  and  $y$  directions, respectively (Meirovitch L, 1986)



**Figure 1:** Cantilever bare square plate



**Figure 2:** Free-Free bare square plate

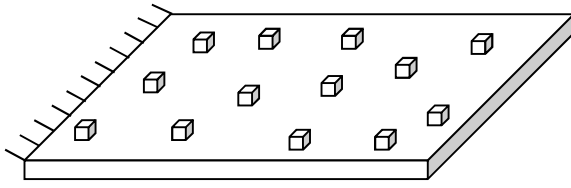
Lagrange's equation results in the equation of motion of the bare plates as given below

$$\ddot{q} + \omega^2 q = 0 \quad (1)$$

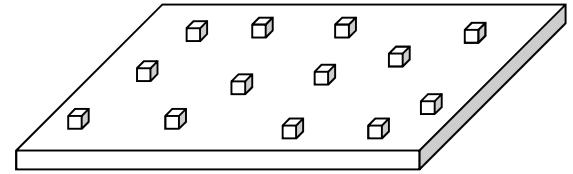
The natural frequencies can then be obtained by eigenvalue analysis (Werner Soedel, 1993).

### 2.2 Uncertain Mass Loaded Plate

Now consider the uncertain mass loaded plate as shown in Figure 3 and Figure 4 for this the equation of motion (2) is (Kessissoglou, N.J et al., 2004)



**Figure3:** Cantilever square plate with mass uncertainty



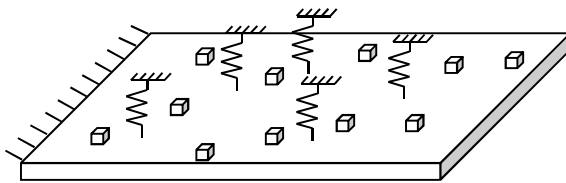
**Figure 4:** Free-Free square plate with mass

$$\ddot{q}_{pq} + \sum_{N_m} \sum_{mn} m_a \ddot{q}_{mn} \psi_{mn}(x_m) \psi_{pq}(x_m) + \omega_{pq}^2 q_{pq} = 0 \quad (2)$$

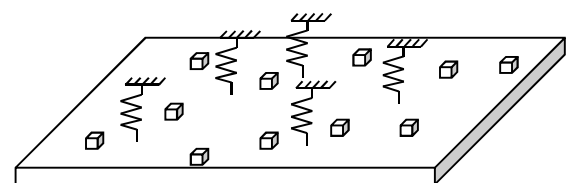
Where;  $N_m$  is number of point masses,  $\psi_{mn}(x)$  are the mass-normalized eigenfunctions and  $x_m$  correspond to the random locations of the added masses

### 2.3 Uncertain Mass-and-Spring-Loaded Plate

Now consider a mass-and-spring-loaded plate as shown in Figure 5 and Figure 6 for this the equation of motion is (Nicole J. Kessissoglou et al., 2009)



**Figure 5:** Cantilever plate with mass & stiffness



**Figure6:** Free-Free plate with mass & stiffness

uncertainty

uncertainty

$$\ddot{q}_{pq} + \sum_{N_m} \sum_{mn} m_a \ddot{q}_{mn} \psi_{mn}(x_m) \psi_{pq}(x_m) + \sum_{N_k} \sum_{mn} k q_{mn} \psi_{mn}(x_k) \psi_{pq}(x_k) + \omega_{pq}^2 q_{pq} = 0 \quad (3)$$

Where;  $N_m$  number of point masses and  $N_k$  springs to ground (of stiffness  $k$ ),  $\psi_{mn}(x)$  are the mass-normalized eigenfunctions, and  $x_m$ ,  $x_k$  respectively correspond to the random locations of the added masses and springs.

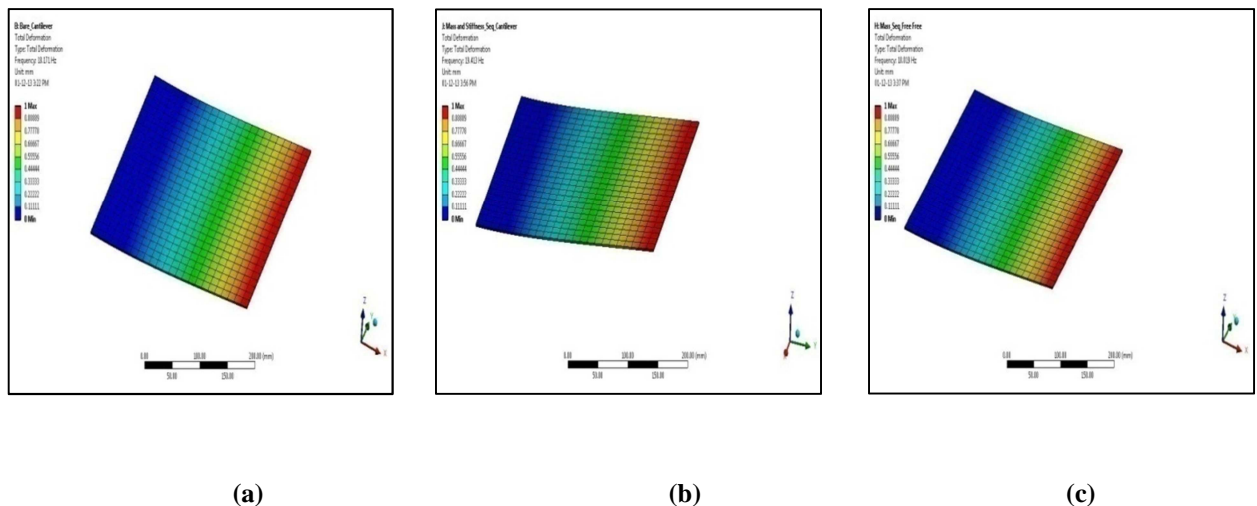
### 3. Dynamic analysis of rectangular plate with and without uncertainty

A square plate of dimension 304.8mm×304.8mm×2mm of steel material with properties of  $\rho=7.86 \times 10^{-9}$ ,  $\nu=0.3$ ,  $Y=2e5$  MPa was created. Then applied all material properties and cantilever and free-free boundary condition. Selecting the element type quadratic shell element 181. SHELL181 is suitable for analyzing thin to moderately-thick shell structures. It is a four-node element with six degrees of freedom at each node translations in the  $x$ ,  $y$ , and  $z$  directions, and rotations about the  $x$ ,  $y$ , and  $z$ -axes. The degenerate triangular option should only be used as filler elements in mesh generation. Shell 181 is well-suited for linear, large rotation, and/or large strain nonlinear applications. Change in shell thickness is accounted for in nonlinear analyses (ANSYS R14.5, 2013).

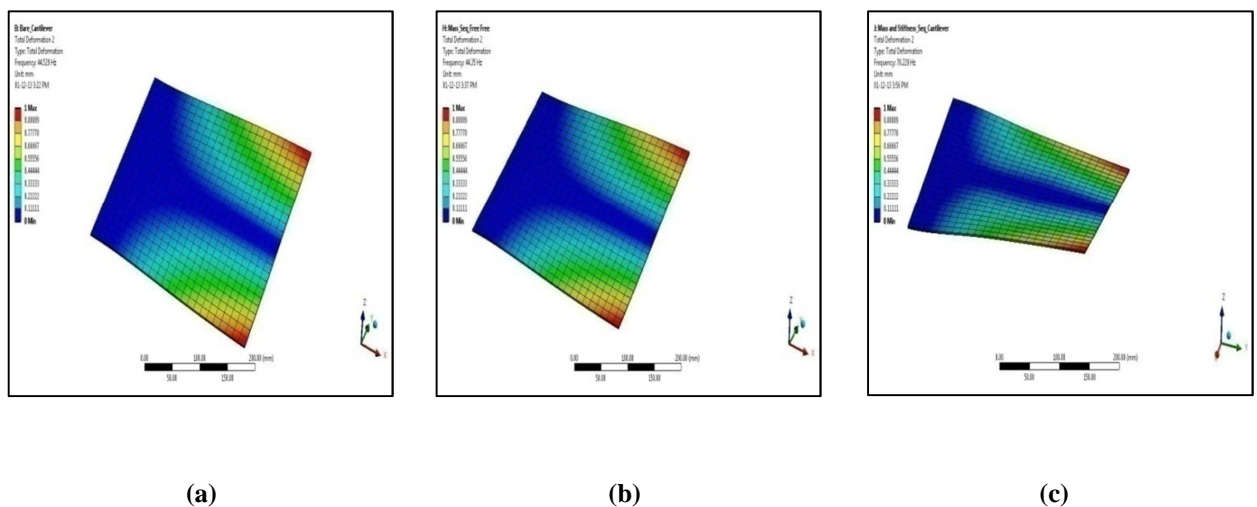
Total mass of plate is  $1.5 \times 10^{-3}$  tones. Mass uncertainty is taken 2% of total mass (Sondipon et al., 2007). Number of masses taken is 13. They are sequentially placed on a plate. For uncertain masses 'mass 21 element' is taken which has three degree of freedom at each node (ANSYS R14.5, 2013).

#### 4. Effect of uncertain parameters on the natural frequency and mode shape

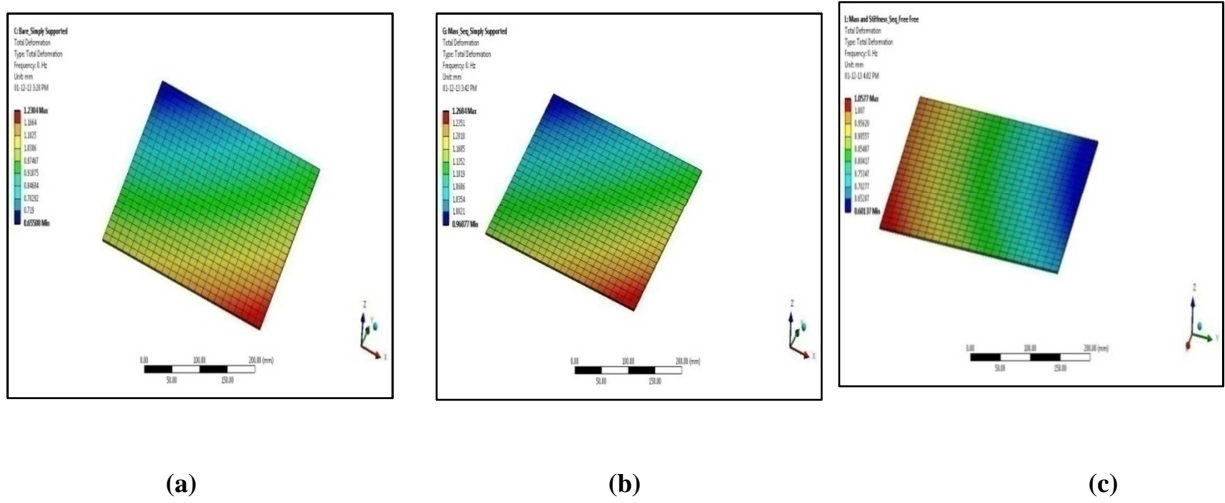
The frequency range is taken from 0 Hz to 3000Hz. Plates are excited for 30 modes. The solver used is Block Lanczos (ANSYS R14.5, 2013). It gives 30 natural frequency values for bare plate, mass uncertainty plate and combination of mass and stiffness uncertainty plate shown in the Table 1 for cantilever and Table 2 shows for free-free condition plate. Mode shape in Figure 7 and Figure 8 for cantilever plate and Figure 9 and Figure 10 for Free-Free condition plate. Natural frequency get reduces due to mass uncertainty and its percentage decreased by -0.84% is shown in Table 1. Due to stiffness uncertainty natural frequency very prominently increases for combination of mass and stiffness by 6.84%. In Table 2 first six natural frequency values are approximately zero so at seventh mode its value is decrease by 0.25% and very prominently increases for combination of mass and stiffness by 76.17%. These are the effects of uncertainty on the natural frequency. In the first mode there is no change in mode shape obtained as shown in Figure 7. But in the second mode as shown in Figure 8 due to mass uncertainty mode shape is changed and due to stiffness phase is changed and same pattern is obtained for Figure 9 and Figure 10. These effects of uncertainty on the mode shapes were found. Figure 11 and Figure 12 showing frequency deviations of bare plate with uncertain parameter contain plate for cantilever and free-free condition respectively.



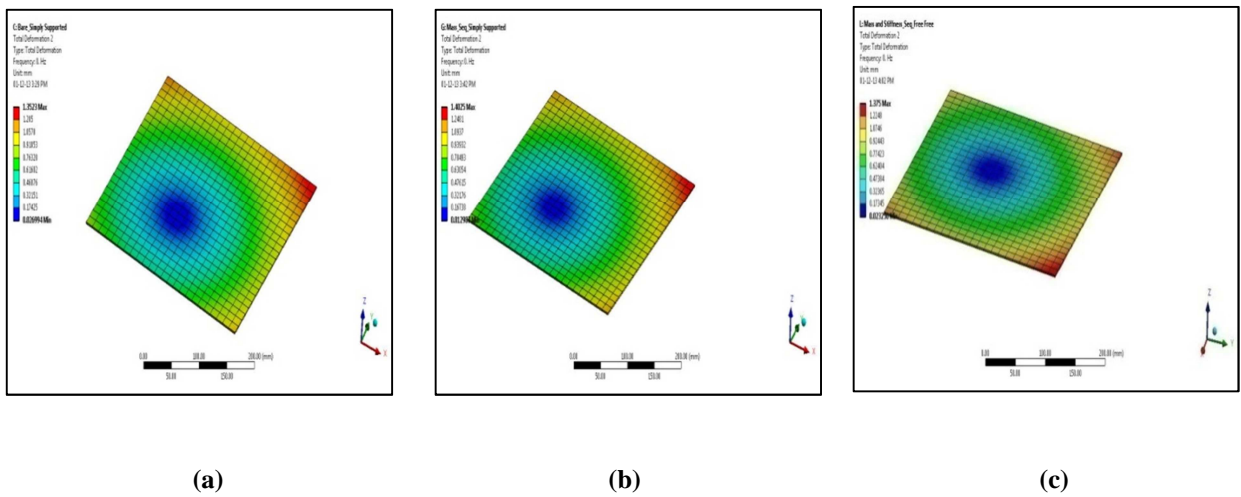
**Figure 7:** First mode shape of Cantilever plate: (a) bare plate, (b) mass uncertainty, (c) mass and stiffness uncertainty



**Figure 8:** Second mode shape of Cantilever plate: (a) bare plate, (b) mass uncertainty, (c) mass and stiffness uncertainty



**Figure 9:** First mode shape of Free-Free plate: (a) bare plate, (b) mass uncertainty, (c) mass and stiffness uncertainty



**Figure 10:** Second mode shape of Free-Free plate: (a) bare plate, (b) mass uncertainty, (c) mass and stiffness uncertainty

Cantilever Condition						
Mode No.	Bare Plate (Hz)	Mass Uncertainty (Hz)	% Error	Mass And Stiffness Uncertainty (Hz)	% Error	
1	18.17	18.02	-0.84	19.41	6.84	
2	44.53	44.35	-0.40	70.23	57.72	
3	111.81	110.76	-0.94	141.06	26.16	

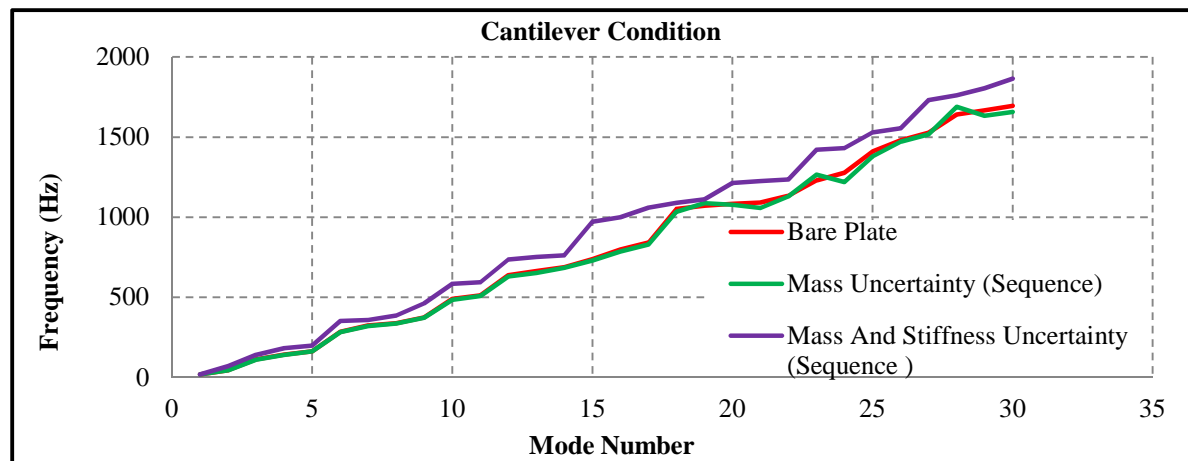
4	142.69	141.02	-1.17	182.33	27.78
5	162.49	161.80	-0.42	199.43	22.73
6	284.57	281.79	-0.98	352.80	23.98
7	324.35	320.36	-1.23	358.96	10.67
8	338.57	336.14	-0.72	385.60	13.89
9	375.22	372.60	-0.70	462.00	23.13
10	489.99	485.07	-1.00	584.46	19.28
11	512.35	507.96	-0.86	594.31	16.00
12	637.38	629.38	-1.26	735.49	15.39
13	663.81	651.57	-1.84	751.09	13.15
14	688.63	683.22	-0.79	761.87	10.64
15	737.69	730.50	-0.97	971.67	31.72
16	797.53	786.28	-1.41	998.99	25.26
17	840.99	830.12	-1.29	1059.90	26.03
18	1051.80	1034.20	-1.67	1088.80	3.52
19	1071.70	1087.30	1.46	1111.20	3.69
20	1084.20	1077.90	-0.58	1214.00	11.97
21	1090.80	1057.40	-3.06	1224.50	12.26
22	1133.00	1129.50	-0.31	1236.10	9.10
23	1229.70	1265.60	2.92	1421.70	15.61
24	1277.70	1218.60	-4.63	1430.80	11.98
25	1408.30	1378.50	-2.12	1529.90	8.63
26	1480.00	1471.80	-0.55	1554.70	5.05
27	1526.90	1516.60	-0.67	1729.90	13.29
28	1640.60	1689.90	3.00	1760.30	7.30
29	1666.90	1632.60	-2.06	1804.50	8.25
30	1695.00	1656.50	-2.27	1864.90	10.02

Table 1: Natural frequency value for 30 mode shapes of Cantilever plate

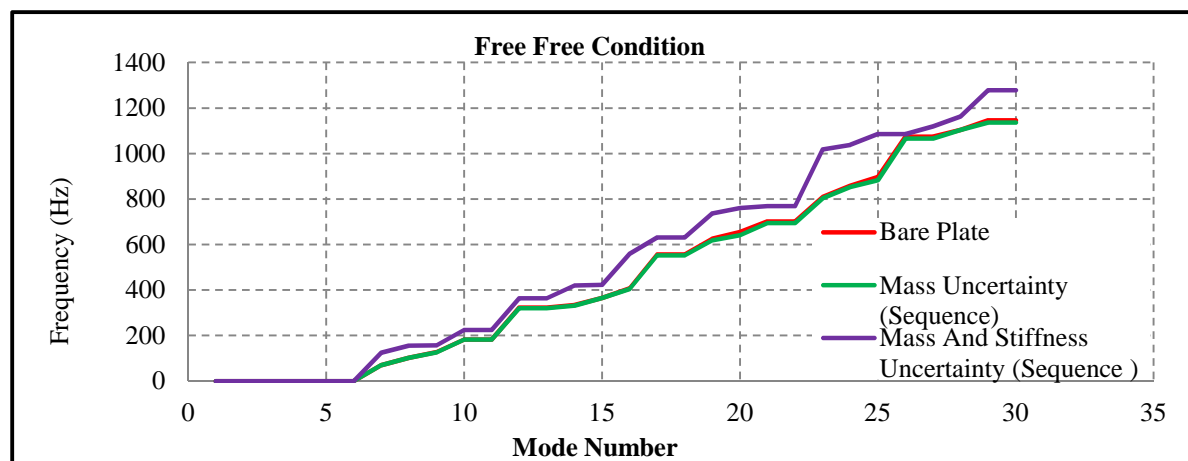
Table 2: Natural frequency value for 30 mode shapes of Free-Free condition plate

<b>Free Free Condition</b>					
<b>Mode No.</b>	<b>Bare Plate (Hz)</b>	<b>Mass Uncertainty (Hz)</b>	<b>% Error</b>	<b>Mass And Stiffness Uncertainty (Hz)</b>	<b>% Error</b>
1	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0000	0.0003	0.0000	0.0000	0.0000
4	0.0004	0.0003	-23.8266	0.0003	-28.5404
5	0.0005	0.0004	-21.3146	0.0004	-9.9876
6	0.0006	0.0005	-18.8317	0.0005	-14.0490
7	70.51	70.33	-0.25	124.21	76.17
8	102.76	102.57	-0.18	155.62	51.44
9	127.41	125.51	-1.49	157.35	23.50
10	182.50	182.06	-0.24	224.71	23.13
11	182.50	182.06	-0.24	224.71	23.13
12	322.81	319.60	-0.99	364.28	12.85
13	322.81	319.60	-0.99	364.28	12.85
14	334.50	331.60	-0.87	420.16	25.61
15	364.93	364.93	0.00	422.71	15.83
16	407.65	404.05	-0.88	559.11	37.15
17	556.11	552.29	-0.69	630.80	13.43
18	556.11	552.29	-0.69	630.80	13.43
19	625.21	618.58	-1.06	737.02	17.88
20	654.49	641.43	-2.00	759.90	16.11
21	701.04	694.69	-0.91	768.94	9.69
22	701.04	694.69	-0.91	768.94	9.69
23	808.13	802.56	-0.69	1017.60	25.92
24	857.84	851.51	-0.74	1038.10	21.01

25	897.32	883.43	-1.55	1085.20	20.94
26	1074.70	1065.30	-0.87	1085.20	0.98
27	1074.70	1065.30	-0.87	1119.40	4.16
28	1103.50	1103.50	0.00	1162.80	5.37
29	1146.70	1136.30	-0.91	1278.90	11.53
30	1146.70	1136.30	-0.91	1278.90	11.53



**Figure 11:** Graph showing frequency deviations of bare plate with uncertain parameter contain plate.



**Figure 12:** Graph showing frequency deviations of bare plate with uncertain parameter contain plate.

## 5. Conclusions

By adding the uncertain mass, we are getting the different dynamic characteristic of plate for the same boundary condition. Though, the amount of uncertain mass is very low still it has influence on the structure. First frequency of the plate has been shift from 18.17 Hz to 18.02 Hz in case of cantilever and 70.51Hz to 70.33Hz for Free- Free boundary condition. It is clearly showing the effect of addition of uncertain mass. This trend of decreasing the plate frequencies are continuing for all the modes.



If we compare the characteristics of bare plate with that of plate with uncertain for the first 30 modes, there is clear deviation in frequency shift which may affect the working condition of entire structure. In case of uncertain mass in sequence pattern there is a deviation in fundamental mode which may not be acceptable in case precise instrumentation or precision manufacturing systems.

Addition of uncertain mass to the bare plate not only shift its frequencies but it also affects the mode shapes of plate. By adding an uncertain mass to the plate structure, apart from fundamental mode, most of the modes are losing their pattern. Irrespective of the frequency shift, the changing vibrating behavior of plate itself is a critical and unavoidable issue.

Addition of the uncertain stiffness dominantly changes dynamic characteristic of plate for the same boundary conditions. First frequency of the plate has been shift from 18.17 Hz to 19.41 Hz in this boundary condition having the deviation of 6.84% in fundamental mode and in free- free condition in seventh mode it is shifted from 70.51Hz to 124.21Hz and with a deviation of 76.71% which may not be acceptable.

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