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Solving Some Integrals with Maple

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Abstract

This study uses the mathematical software Maple for the auxiliary tool to evaluate two types of integrals. We can obtain the Fourier series expansions of these two types of integrals by using Euler's formula, generalized DeMoivre's formula and integration term by term. In addition, we provide two integrals to do calculation practically. The research methods adopted in this study involved finding solutions through manual calculations and verifying these solutions by using Maple. This type of research method not only allows the discovery of calculation errors, but also helps modify the original directions of thinking from manual and Maple calculations. Therefore, Maple provides insights and guidance regarding problem-solving methods.

***Keywords**:* integrals, Fourier series expansions, Euler's formula, generalized DeMoivre's formula, integration term by term, Maple.

1. INTRODUCTION

As information technology advances, whether computers can become comparable with human brains to perform abstract tasks, such as abstract art similar to the paintings of Picasso and musical compositions similar to those of Beethoven, is a natural question. Currently, this appears unattainable. In addition, whether computers can solve abstract and difficult mathematical problems and develop abstract mathematical theories such as those of mathematicians also appears unfeasible. Nevertheless, in seeking for alternatives, we can study what assistance mathematical software can provide. This study introduces how to conduct mathematical research using the mathematical software Maple. The main reasons of using Maple in this study are its simple instructions and ease of use, which enable beginners to learn the operating techniques in a short period. By employing the powerful computing capabilities of Maple, difficult problems can be easily solved. Even when Maple cannot determine the solution, problem-solving hints can be identified and inferred from the approximate values calculated and solutions to similar problems, as determined by Maple. For this reason, Maple can provide insights into scientific research. Inquiring through an online support system provided by Maple or browsing the Maple website (www.maplesoft.com) can facilitate further understanding of Maple and might provide unexpected insights. For the instructions and operations of Maple, [1]-[7] can be adopted as references.

In calculus and engineering mathematics courses, we learnt many methods to solve the integral problems, including change of variables method, integration by parts method, partial fractions method, trigonometric substitution method, and so on. In this paper, we study the evaluation of the following two types of indefinite integrals which are not easy to obtain their answers by using the methods mentioned above.

$$\int e^{\lambda \cos x} \cdot \sin[(r+1)x + \lambda \sin x]dx \tag{1}$$

$$\int e^{\lambda \cos x} \cdot \cos[(r+1)x + \lambda \sin x]dx \tag{2}$$

Where r, λ are real numbers, r is not a negative integer, and $-\pi < x \le \pi$. We can obtain the Fourier series expansions of these two types of indefinite integrals by using Euler's formula, generalized DeMoivre's formula and integration term by term; these are the main results of this paper (i.e., Theorems 1, 2). As for the study of related integral problems can refer to [8]-[16]. On the other hand, we provide two integrals to do calculation practically. The research methods adopted in this study involved finding solutions through manual calculations and verifying these solutions by using Maple. This type of research method not only allows the discovery of calculation errors, but also helps modify the original directions of thinking from manual and Maple calculations. For this reason, Maple provides insights and guidance regarding problem-solving methods.

2. Main Results

Firstly, we introduce two formulas used in this study.

Euler's formula.

$e^{ix} = \cos x + i \sin x$, where x is any real number.

Generalized DeMoivre's formula ([17, p29]).

$(\cos x + i \sin x)^r = \cos rx + i \sin rx$, where r, x are real numbers, $-\pi < x \le \pi$.

Next, we introduce an important theorem used in this paper.

Integration term by term ([17, p269]).

Suppose $\{g_n\}_{n=0}^{\infty}$ is a sequence of Lebesgue integrable functions defined on an inteval I. If $\sum_{n=0}^{\infty}\int_I |g_n|$ is convergent, then $\int_I \sum_{n=0}^{\infty} g_n = \sum_{n=0}^{\infty}\int_I g_n$.

Before deriving the major results in this study, we need a lemma.

Lemma. Suppose r, λ are real numbers, r is not a negative integer, z is a complex number, and C is a constant. Then the indefinite integral

$$\int z^r e^{\lambda z} dz = \sum_{n=0}^{\infty} \frac{\lambda^n}{n!(n+r+1)} z^{n+r+1} + C \tag{3}$$

Proof.

$$\int z^r e^{\lambda z} dz = \int z^r \cdot \sum_{n=0}^{\infty} \frac{1}{n!}(\lambda z)^n dz$$

$$= \int \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} z^{n+r} dz$$

$$= \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} \int z^{n+r} dz \quad \text{(by integration term by term)}$$

$$= \sum_{n=0}^{\infty} \frac{\lambda^n}{n!(n+r+1)} z^{n+r+1} + C \qquad \blacksquare$$

The following is the first major result of this study; we determine the Fourier series expansion of indefinite integral (1).

Theorem 1. Assume r, λ are real numbers, and r is not a negative integer. Then the indefinite integral

$$\int e^{\lambda \cos x} \cdot \sin[(r+1)x + \lambda \sin x] dx = -\sum_{n=0}^{\infty} \frac{\lambda^n}{n!(n+r+1)} \cos[(n+r+1)x] + C \tag{4}$$

For all real numbers x satisfy $-\pi < x \le \pi$.

Proof. Taking $z = e^{ix}$ into (3), we obtain

$$i \cdot \int (e^{ix})^r e^{\lambda e^{ix}} e^{ix} dx = \sum_{n=0}^{\infty} \frac{\lambda^n}{n!(n+r+1)} (e^{ix})^{n+r+1} + C$$

$$\Rightarrow i \cdot \int e^{i(r+1)x} e^{\lambda(\cos x + i \sin x)} dx = \sum_{n=0}^{\infty} \frac{\lambda^n}{n!(n+r+1)} e^{i(n+r+1)x} + C$$

(By generalized DeMoivre's formula)

$$\Rightarrow i \cdot \int e^{\lambda \cos x} e^{i[(r+1)x+\lambda \sin x]} dx = \sum_{n=0}^{\infty} \frac{\lambda^n}{n!(n+r+1)} [\cos(n+r+1)x + i \sin(n+r+1)x] + C$$

(by Euler's formula) (5)

Using the equal of the real parts of both sides of (5), we have

$$\int e^{\lambda \cos x} \cdot \sin[(r+1)x + \lambda \sin x] dx = -\sum_{n=0}^{\infty} \frac{\lambda^n}{n!(n+r+1)} \cos[(n+r+1)x] + C$$

for all real numbers x satisfy $-\pi < x \leq \pi$ ■

Next, we obtain the Fourier series expansion of indefinite integral (2).

Theorem 2. If the assumptions are the same as Theorem 1, then the indefinite integral

$$\int e^{\lambda \cos x} \cdot \cos[(r+1)x + \lambda \sin x] dx = \sum_{n=0}^{\infty} \frac{\lambda^n}{n!(n+r+1)} \sin[(n+r+1)x] + C \quad (6)$$

For all real numbers x satisfy $-\pi < x \leq \pi$.

Proof. By the equal of the imaginary parts of both sides of (5), we have

$$\int e^{\lambda \cos x} \cdot \cos[(r+1)x + \lambda \sin x] dx = \sum_{n=0}^{\infty} \frac{\lambda^n}{n!(n+r+1)} \sin[(n+r+1)x] + C$$

for all real numbers x satisfy $-\pi < x \leq \pi$ ■

3. Examples

In the following, aimed at the two types of integrals in this study, we propose two integrals and use Theorems 1, 2 to determine their Fourier series expansions. On the other hand, we use Maple to calculate the approximations of related definite integrals and their solutions for verifying our answers.

Example 1. Using Theorem 1, we obtain the indefinite integral

$$\int e^{(1/3)\cos x} \cdot \sin[(\sqrt{2}+1)x + (1/3)\sin x] dx = -\sum_{n=0}^{\infty} \frac{(1/3)^n}{n!(n+\sqrt{2}+1)} \cos[(n+\sqrt{2}+1)x] + C \quad (7)$$

for all real numbers x satisfy $-\pi < x \leq \pi$ (the case of $r = \sqrt{2}, \lambda = 1/3$ in Theorem 1).

Therefore, we can determine the following definite integral

$$\int_{-\pi/2}^{\pi/3} e^{(1/3)\cos x} \cdot \sin[(\sqrt{2}+1)x + (1/3)\sin x] dx$$

$$= -\sum_{n=0}^{\infty} \frac{(1/3)^n}{n!(n+\sqrt{2}+1)}\{\cos[(n+\sqrt{2}+1)\pi/3] - \cos[(n+\sqrt{2}+1)\pi/2]\} \quad (8)$$

Next, we use Maple to verify the correctness of (8).

>evalf (int (exp (cos(x)/3)*sin ((sqrt (2) +1)*x+sin(x)/3), x=-Pi/2..Pi/3),22);

0.1662169095583142069991

>evalf(-sum((1/3)^n/(n!*(n+sqrt(2)+1))*(cos((n+sqrt(2)+1)*Pi/3)-cos((n+sqrt(2)+1)*Pi/2)),n=0..infinity),22);

$0.1662169095583142069785 - 7 \cdot 10^{-21}\, I$

The above answer obtained by Maple appears I ($=\sqrt{-1}$), it is because Maple calculates by using special functions built in. The imaginary part is very small, so can be ignored.

Example 2. By Theorem 2, we have the indefinite integral

$$\int e^{\sqrt{5}\cos x} \cdot \cos[(1/7)x + \sqrt{5}\sin x]dx = \sum_{n=0}^{\infty} \frac{\sqrt{5}^n}{n!(n+1/7)}\sin[(n+1/7)x] + C \quad (9)$$

for all real numbers x satisfy $-\pi < x \le \pi$ (the case of $r = -6/7, \lambda = \sqrt{5}$ in Theorem 2).

Thus, we can evaluate the following definite integral

$$\int_{-3\pi/4}^{\pi/2} e^{\sqrt{5}\cos x} \cdot \cos[(1/7)x + \sqrt{5}\sin x]dx$$

$$= \sum_{n=0}^{\infty} \frac{\sqrt{5}^n}{n!(n+1/7)}\{\sin[(n+1/7)\pi/2] + \sin[(3n+3/7)\pi/4]\} \quad (10)$$

We also use Maple to verify the correctness of (10).

>evalf (int (exp (sqrt (5)*cos(x))*cos ((1/7)*x+sqrt (5)*sin(x)), x=-3*Pi/4..Pi/2),22);

5.250651103931792117654

>evalf(sum((sqrt(5))^n/(n!*(n+1/7))*(sin((n+1/7)*Pi/2)+sin((3*n+3/7)*Pi/4)),n=0..infinity),22);

$5.250651103931792117655 + 1.373275422506790546221 \cdot 10^{-21}\, I$

The above answer obtained by Maple also appears I, the imaginary part is very small, so can be ignored.

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4. Conclusion

From the above discussion, we know the Euler's formula, the generalized DeMoivre's formula and the integration term by term play significant roles in the theoretical inferences of this study. In fact, the applications of them are extensive, and can be used to easily solve many difficult problems; we endeavour to conduct further studies on related applications. On the other hand, Maple also plays a vital assistive role in problem-solving. In the future, we will extend the research topic to other calculus and engineering mathematics problems and solve these problems by using Maple. These results will be used as teaching materials for Maple on education and research to enhance the connotations of calculus and engineering mathematics.

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