

EFFECTS OF CRACK ON MODAL FREQUENCY OF CANTILEVER BEAM

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Abstract

Finite Element Method is used to determine the natural frequency of beam whereas the experimentation is performed using Fast Fourier Transform analyser. In experimentation, cantilever beam with crack are considered. The experimental results are validated with the results of FEM software. This formulation can be extended for various boundary conditions as well as varying cross sectional areas. A crack in a structural member introduces local flexibility that would affect vibration response of the structure. i.e., a crack causes a reduction in the stiffness and an increase in the damping of the structure. These changes of physical properties cause a reduction in the natural frequencies and a deviation in the mode shape. Therefore it is possible to predict the crack depth and crack location by measuring changes in the vibration parameters. Changes in the natural frequencies are more often considered than deviation of mode shapes, since frequencies can be measured more easily than mode shapes, and they are less seriously affected by experimental errors. This property may be used to detect existence of a crack together with its location and depth in the structural member. In this analysis the natural frequencies obtained from experimental analysis and finite element analysis are used to obtain the crack location and crack depth.

Keywords: Crack, FEM, FFT, Modal Analysis, Natural Frequency.

1. Introduction

Many structural applications worldwide have been in use form many years. Their failure could lead to tragic consequences and therefore structures have regular costly inspections. During the last decades vibration based damage detection methods have attracted the most attention due to their simplicity for implementation. The presence of crack in structure changes its dynamic characteristics [2]. The change is characterized by change in modal parameters like modal frequencies, modal value and mode shapes associated with each modal frequency [13]. It also alters the structural parameters like mass, damping matrix, stiffness matrix and flexibility matrix of

structure [13]. The vibration technique utilizes one or more of these parameters for crack detection. The frequency reduction in cracked beam is not due to removal of mass from beam, indeed the reduction in mass would increase natural frequency [13]. But reduction in natural frequency is observed due to removal of material which carries significant stresses when defect is a narrow crack or notch. It reduces the stiffness of structure and natural frequency [3]. Due to presence of crack there is local influence which results from reduction area of cross section where it is located [29]. Finite Element Analysis is powerful tool which gives the reasonably accurate results for complicated structure. The present study is based on observation of changes in natural frequency.

2. Experimental set up

The instruments used for experimental analysis are Fast Fourier Transform (FFT) analyser, accelerometer, impact hammer and related accessories. The accelerometer is mounted on the beam using mounting clips. The accelerometer is mounted near the crack to capture the correct signal. The impact hammer is used to excite the beam whose frequency response function has to be captured. For every test, the location of impact of impact hammer is kept constant. The beam is tapped gently with the impact hammer. The experiments are performed on mild steel (IS 2062 : 2011 E250 A) beams with cantilever boundary conditions with crack of different depths at different locations. The properties of mild steel are, Young's modulus (E) 2.1×10^{11} N/ m^2 , density (ρ) 7860 N/ m^3 and Poisson's ratio 0.3. Specimen beams under consideration have circular cross section area. For solid cantilever beam the cross sectional diameter is 40 mm. The geometry of beams is as shown in Figure 1. Crack depth is represented in terms of (a/d) ratio where a is depth of crack and d is diameter of beam and crack location is represented in terms of (e) where (e) is ratio of location of crack at distance L_1 from the fixed support to the length of the beam L . The experimental setup is as shown in Figure 2. The aim of experimental analysis is to verify the practical applicability of the theoretical method developed. For the beam with single cracks, transverse and open cracks are considered. Initially, the natural frequency of uncrack beam is found out. Hairline crack is generated to simulate the actual crack in the working components. There after depth of crack is increased. The change in natural frequency due to the crack is monitored. Table I shows the natural frequencies of simply supported beam with single crack.

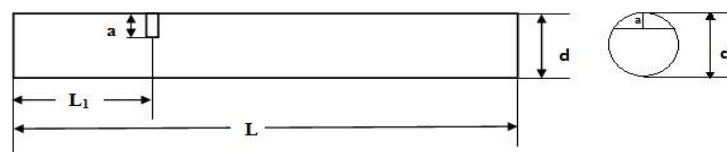


Figure 1: Geometry of beam with single crack

**Figure 2:** Experimental setup

3. Computational work

The solid cantilever beam of the cross sectional diameter 40 mm was prepared from mild steel. The Finite Element Method (FEM) of an artificially cracked and un-cracked beam had carried out with the help of ANSYS 14.5 package, the cracked beam is modelled as a solid beam. The mesh generation, analysis, display of results, are all performed within ANSYS Workbench 14.5. For the natural frequencies by FEM software refer Table II and all mode shape are as shown in Fig. 5-9.

Table 1: Natural Frequencies of Cantilever Beam by experiments

Crack Size and Location (mm)			Natural Frequency by Experiments (Hz)				
No.	a/d	e	1	2	3	4	5
1	0	0	843.8	2250	4195	6563	9258
2	0.1	0.2159	820.3	2227	4172	6539	9211
3	0.1	0.4319	820.3	2203	4172	6516	9188
4	0.1	0.6494	796.9	2180	4172	6516	9188
5	0.1	0.8654	796.9	2203	4148	6539	9164
6	0.2	0.2159	796.9	2180	4102	6469	9164
7	0.2	0.4319	773.4	2157	4102	6375	9141
8	0.2	0.6494	750	2156	4148	6445	9141
9	0.2	0.8654	726.6	2157	4078	6375	9070
10	0.3	0.2159	773.4	2156	4055	6375	9117
11	0.3	0.4319	726.6	2133	4078	6352	9070
12	0.3	0.6494	703.1	2086	4102	6305	9023
13	0.3	0.8654	656.3	2133	3984	6281	8977
14	0.4	0.2159	750	2133	3984	6305	9023

15	0.4	0.4319	679.7	2086	4031	6258	9000
16	0.4	0.6494	656.3	2039	4078	6234	8953
17	0.4	0.8654	632.8	2086	3914	6188	8906
18	0.5	0.2159	726.6	2086	3914	6281	8977
19	0.5	0.4319	632.8	2063	3961	6211	8977
20	0.5	0.6494	609.4	1992	4055	6188	8930
21	0.5	0.8654	609.4	2039	3867	6070	8859

Table 2: Natural Frequencies of Cantilever Beam by FEM software

Crack Size and Location (mm)			Natural Frequency by FEM (Hz)				
No.	a/d	e	1	2	3	4	5
1	0	0	831.79	2245.8	4196.1	6569.6	9260.5
2	0.1	0.2159	825.12	2225.4	4153.7	6516.4	9199.9
3	0.1	0.4319	822.4	2227	4157.3	6511.6	9197.7
4	0.1	0.6494	825.95	2229.4	4180.6	6534.2	9214.9
5	0.1	0.8654	827.83	2233.6	4168.6	6521.1	9192.5
6	0.2	0.2159	825.08	2211.3	4108.9	6481.5	9190.2
7	0.2	0.4319	812	2219.9	4125.9	6460.9	9175.8
8	0.2	0.6494	815.73	2198.5	4178.3	6474.7	9148.1
9	0.2	0.8654	827.35	2224.4	4128.5	6429.1	9066
10	0.3	0.2159	824.86	2184	4040.3	6428.6	9169.4
11	0.3	0.4319	792.52	2205.9	4072.3	6375.6	9135.7
12	0.3	0.6494	796.42	2143.8	4172.3	6367	9071.8
13	0.3	0.8654	826.06	2204.9	4046.8	6253.5	8888.2
14	0.4	0.2159	824.08	2133	3959.9	6362.3	9122.1
15	0.4	0.4319	761.13	2181.4	4001.6	6263.2	9079.7
16	0.4	0.6494	762.53	2058.7	4157.9	6191	9018.1
17	0.4	0.8654	823.53	2166.3	3894.7	5970.5	8751.1
18	0.5	0.2159	822.64	2040.6	3892.8	6293.7	9028.6
19	0.5	0.4319	716.04	2139.5	3930.3	6142.8	9012.6
20	0.5	0.6494	709.16	1946.6	4127.3	5950	8995.2
21	0.5	0.8654	818.82	2091.6	3652.9	5628.3	8703.4

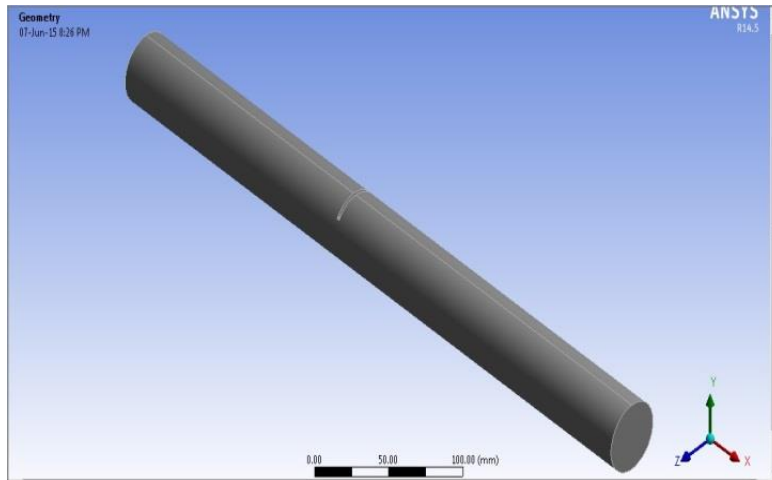


Figure 3: Cracked beam 8mm depth at 200mm from the fixed end

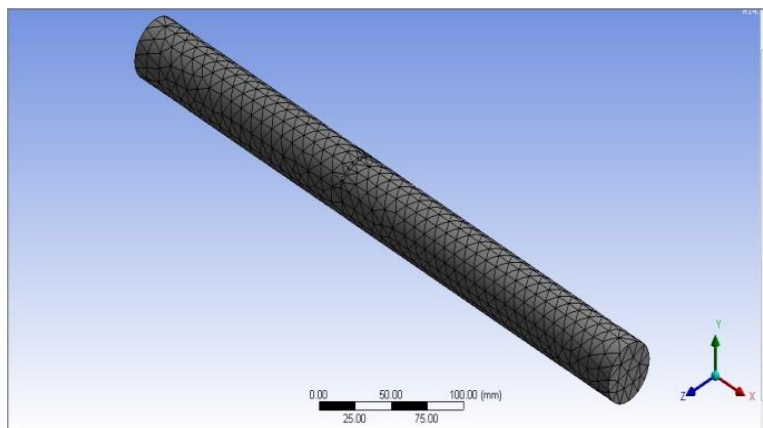


Figure 4: Meshing of cracked beam

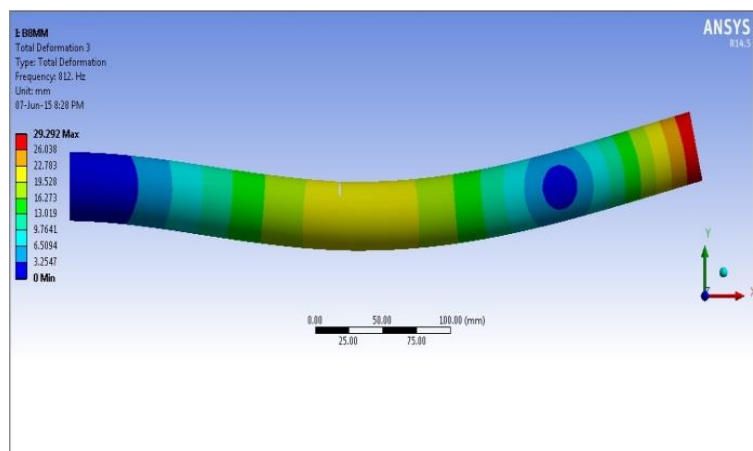


Figure 5: First mode shape of cracked beam

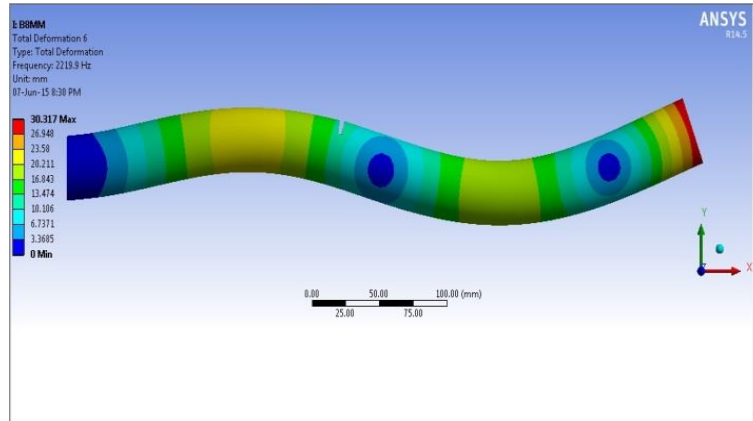


Figure 6: Second mode shape of cracked beam

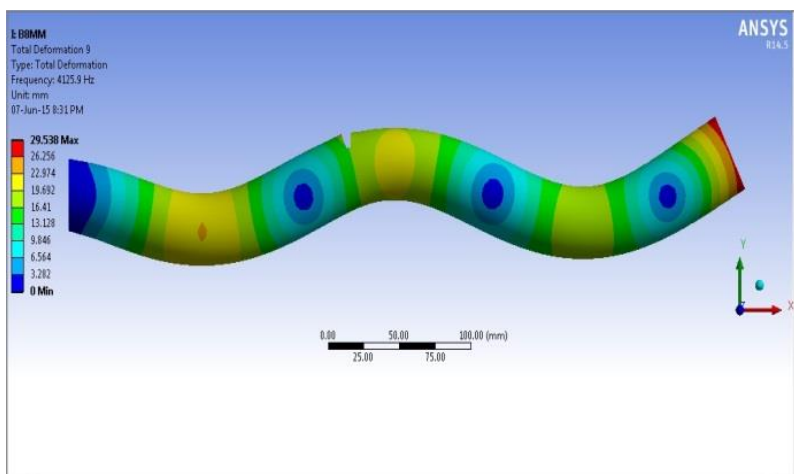


Figure 7: Third mode shape of cracked beam

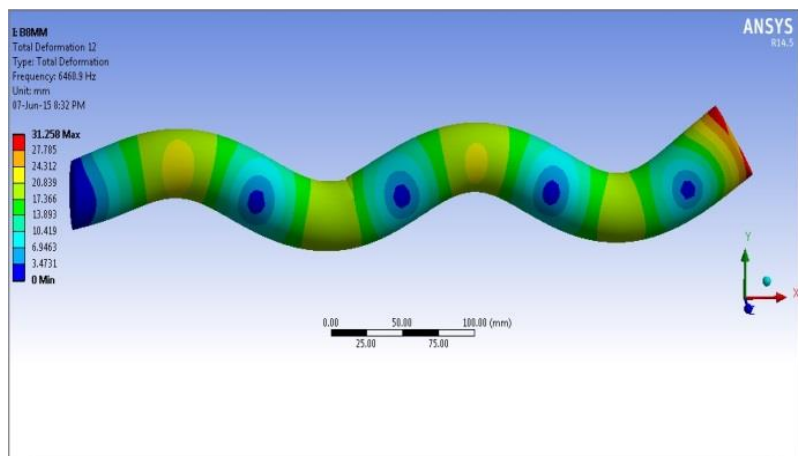
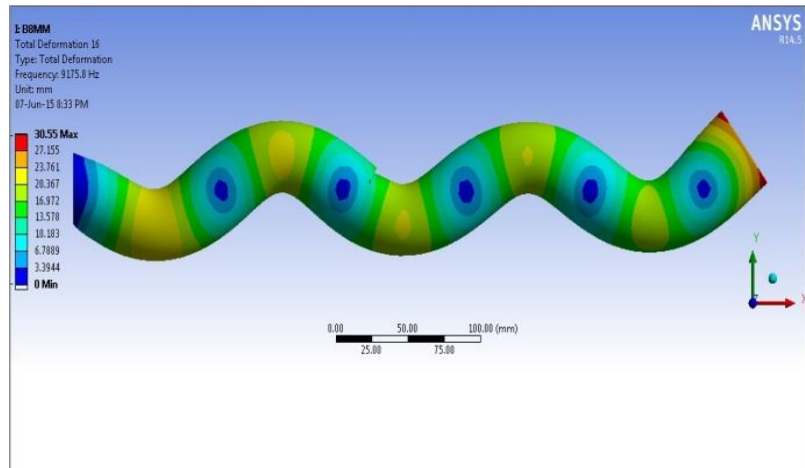


Figure 8: Fourth mode shape of cracked beam**Figure 9:** Fifth mode shape of cracked beam

4. Results

The crack is generated at known location in mild steel beam. The changes in natural frequencies for the uncracked and cracked beams are measured. The predicted values are determined by theoretical and experimental technique. Table I shows the natural frequency values extracted for cantilever beam with single crack determined by using experimentation. Table II shows the natural frequency values extracted for cantilever beam with single crack determined by using FEM software. the results for crack location (e), crack size (a/d) are computed. Table III shows the % reduction in natural frequencies for crack location (e), crack size (a/d) between cracked and uncracked values for cantilever beam with single crack by using experimentation. Table IV shows the % reduction in natural frequencies for crack location (e), crack size (a/d) between cracked and uncracked values for cantilever beam with single crack by using FEM software. It is observed that experimental results have some deviation from the results obtained by FEM as model of structure generated by Finite Element Analysis differs from actual structure. Hence the response of structure in practice differs. The results are close to the actual for finding the crack locations. These results approach to the actual results found by FEM as compared to the experimental findings. The results of crack depth findings are close to the actual depth for large (a/d) ratio as compared to small (a/d) ratio. It is observed because for small (a/d) ratio, the reduction in the stiffness of beam is less as compared to large (a/d) ratio. Due to the high stiffness, the vibrations are damped and natural frequency does not reduce. The readings obtained are used as database for finding the crack location and size from fixed end. Effect of crack depth (a/d) and position (e) on the natural frequency are as shown in fig.10-19.

Table 3: % reduction in Natural Frequencies of Cantilever Beam by experiments

Crack Size and Location (mm)			% Reduction in Natural Frequency by Experiments (Hz)				
No.	a/d	e	1	2	3	4	5
1	0.0	0.00	843.80	2250	4195	6563	9258
2	0.1	0.2159	2.7850	1.0222	0.5483	0.365	0.507
3	0.1	0.4319	2.7850	2.0889	0.5483	0.716	0.756
4	0.1	0.6494	5.5582	3.1111	0.5483	0.716	0.756
5	0.1	0.8654	5.5582	2.0889	1.1204	0.365	1.015
6	0.2	0.2159	5.5582	3.1111	2.2169	1.432	1.015
7	0.2	0.4319	8.3432	4.1333	2.2169	2.864	1.263
8	0.2	0.6494	11.1164	4.1778	1.1204	1.798	1.263
9	0.2	0.8654	13.8895	4.1333	2.7890	2.864	2.030
10	0.3	0.2159	8.3432	4.1778	3.3373	2.864	1.523
11	0.3	0.4319	13.8895	5.2000	2.7890	3.215	2.030
12	0.3	0.6494	16.6746	7.2889	2.2169	3.931	2.538
13	0.3	0.8654	22.2209	5.2000	5.0298	4.296	3.035
14	0.4	0.2159	11.1164	5.2000	5.0298	3.931	2.538
15	0.4	0.4319	19.4477	7.2889	3.9094	4.647	2.786
16	0.4	0.6494	22.2209	9.3778	2.7890	5.013	3.294
17	0.4	0.8654	25.0059	7.2889	6.6985	5.713	3.802
18	0.5	0.2159	13.8895	7.2889	6.6985	4.296	3.035
19	0.5	0.4319	25.0059	8.3111	5.5781	5.363	3.035
20	0.5	0.6494	27.7791	11.466	3.3373	5.713	3.542
21	0.5	0.8654	27.7791	9.3778	7.8188	7.511	4.309

Table 4: % reduction in Natural Frequencies of Cantilever Beam by FEM software

Crack Size and Location (mm)			% Reduction in Natural Frequency by FEM (Hz)				
No.	a/d	e	1	2	3	4	5
1	0.0	0.00	831.79	2245.8	4196.1	6569.6	9260.5
2	0.1	0.2159	0.8019	0.9084	1.0105	0.8098	0.6544
3	0.1	0.4319	1.1289	0.8371	0.9247	0.8829	0.6781
4	0.1	0.6494	0.7021	0.7303	0.3694	0.5388	0.4924
5	0.1	0.8654	0.4761	0.5432	0.6554	0.7382	0.7343
6	0.2	0.2159	0.8067	1.5362	2.0781	1.3410	0.7591
7	0.2	0.4319	2.3792	1.1533	1.6730	1.6546	0.9146
8	0.2	0.6494	1.9308	2.1062	0.4242	1.4445	1.2138
9	0.2	0.8654	0.5338	0.9529	1.6110	2.1386	2.1003
10	0.3	0.2159	0.8331	2.7518	3.7130	2.1462	0.9837

11	0.3	0.4319	4.7211	1.7766	2.9504	2.9530	1.3477
12	0.3	0.6494	4.2523	4.5418	0.5672	3.0839	2.0377
13	0.3	0.8654	0.6889	1.8212	3.5581	4.8116	4.0203
14	0.4	0.2159	0.9269	5.0227	5.6290	3.1554	1.4945
15	0.4	0.4319	8.4949	2.8676	4.6353	4.6639	1.9524
16	0.4	0.6494	8.3266	8.3311	0.9104	5.7629	2.6176
17	0.4	0.8654	0.9930	3.5399	7.1829	9.1193	5.5008
18	0.5	0.2159	1.1000	9.1371	7.2281	4.1996	2.5042
19	0.5	0.4319	13.915	4.7333	6.3345	6.4966	2.6770
20	0.5	0.6494	14.742	13.322	1.6396	9.4313	2.8649
21	0.5	0.8654	1.5593	6.8662	12.945	14.328	6.0159

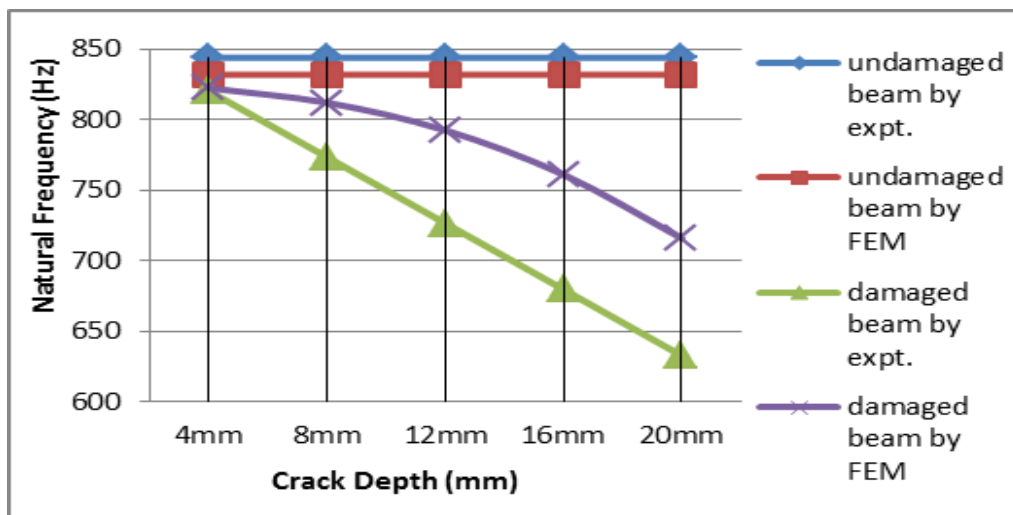


Figure 10: Effect of crack depth on the natural frequency of the first mode shape of a damaged cantilever beam at 200 mm from the fixed end

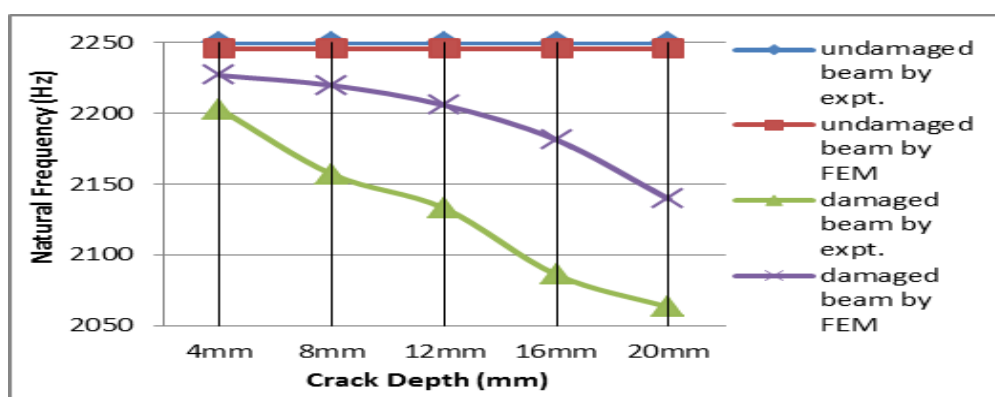


Figure 11: Effect of crack depth on the natural frequency of the second mode shape of a damaged cantilever beam at 200 mm from the fixed end

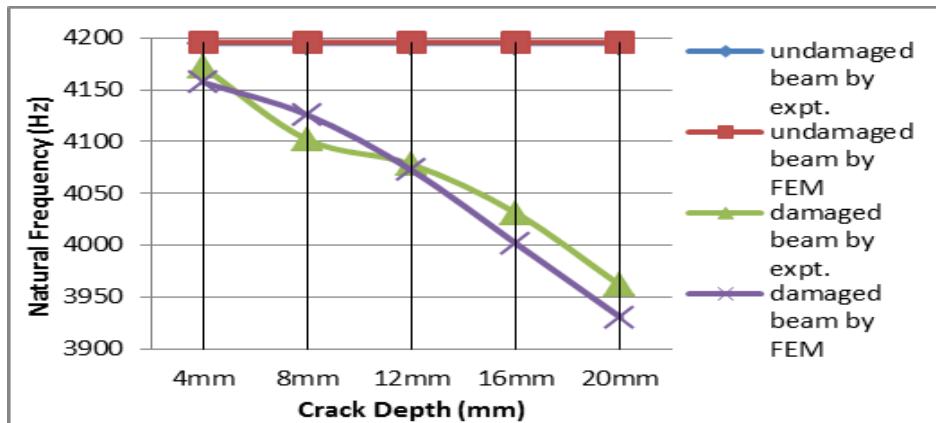


Figure 12: Effect of crack depth on the natural frequency of the third mode shape of a damaged cantilever beam at 200 mm from the fixed end

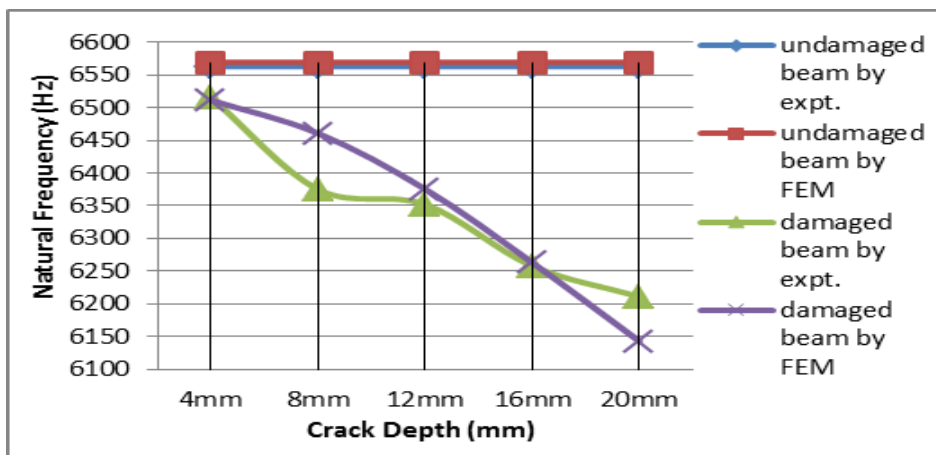


Figure 13: Effect of crack depth on the natural frequency of the fourth mode shape of a damaged cantilever beam at 200 mm from the fixed end

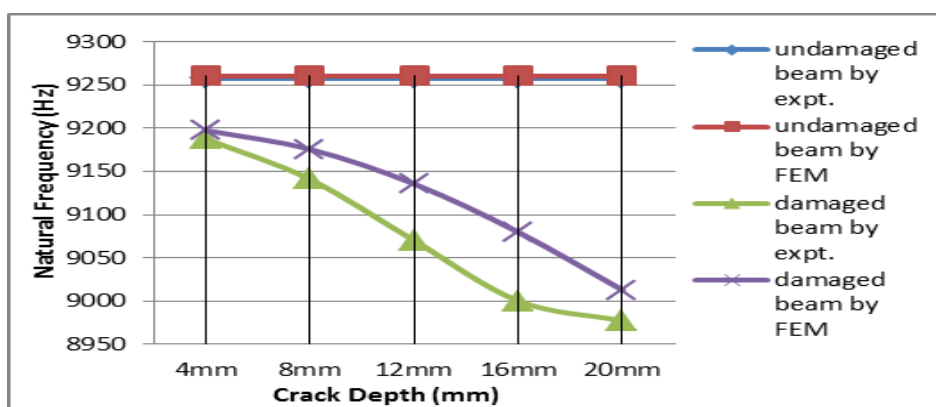


Figure 14: Effect of crack depth on the natural frequency of the fifth mode shape of a damaged cantilever beam at 200 mm from the fixed end

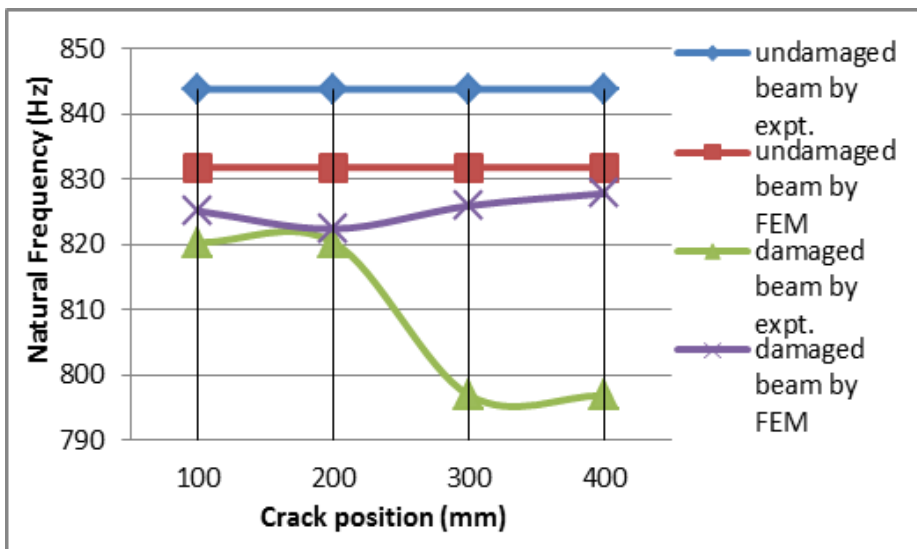


Figure 15: Effect of crack position on the natural frequency of the first mode shape of damaged cantilever beam with 4 mm crack depth from the fixed end

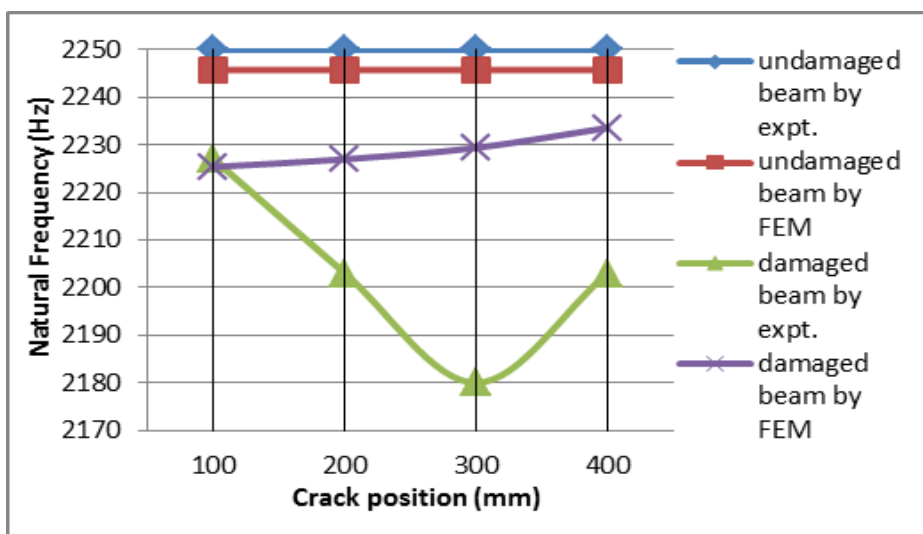


Figure 16: Effect of crack position on the natural frequency of the second mode shape of damaged cantilever beam with 4 mm crack depth from the fixed end

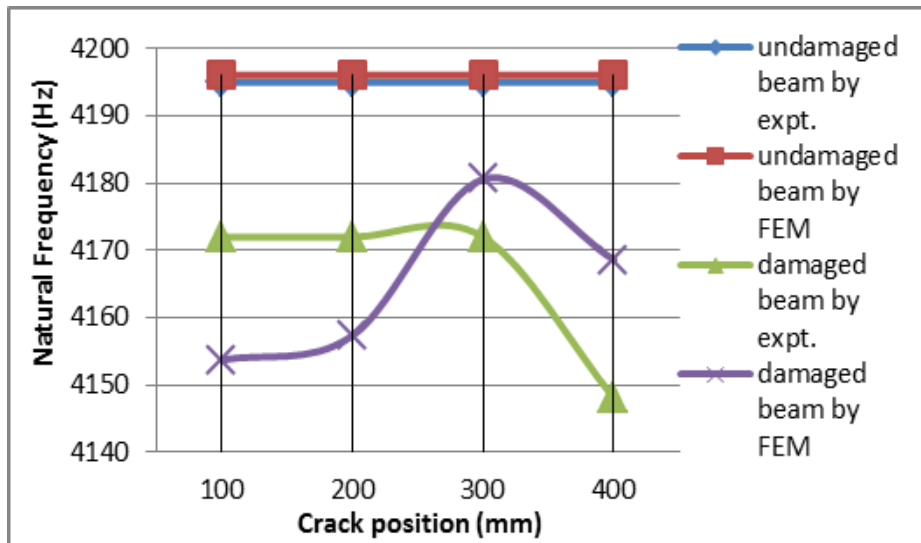


Figure 17: Effect of crack position on the natural frequency of the third mode shape of damaged cantilever beam with 4 mm crack depth from the fixed end

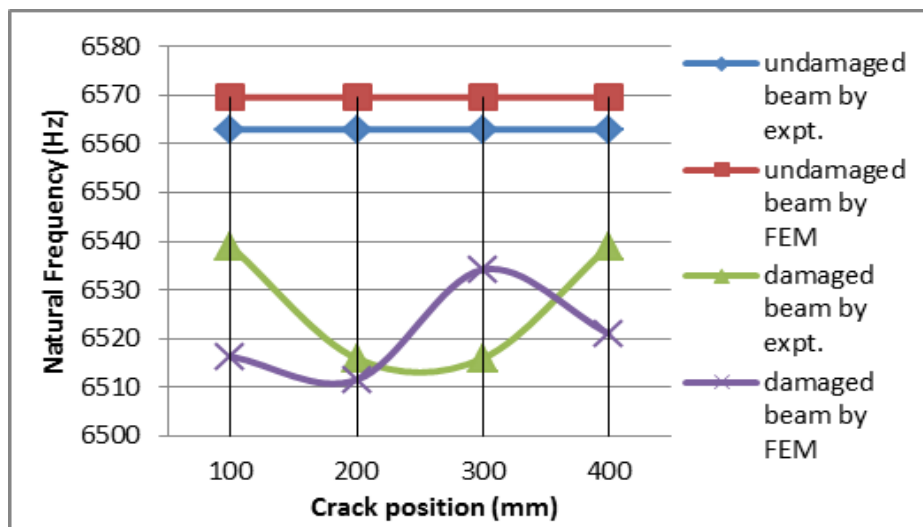


Figure 18: Effect of crack position on the natural frequency of the fourth mode shape of damaged cantilever beam with 4 mm crack depth from the fixed end

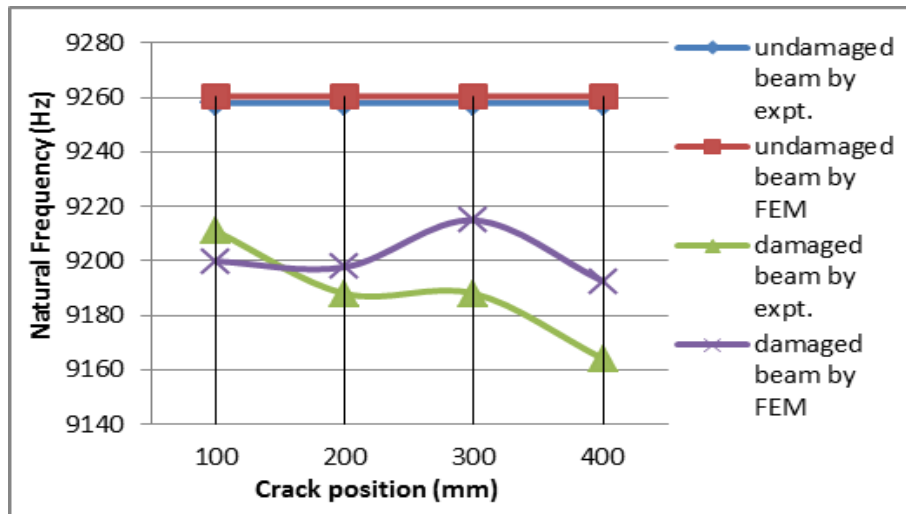


Figure 19: Effect of crack position on the natural frequency of the fifth mode shape of damaged cantilever beam with 4 mm crack depth from the fixed end

5. Conclusion

This work attempts to establish a systematic method of prediction of crack characteristics from measurement of natural frequencies using experimental study, following conclusions can be drawn

1. Natural frequency of the of cracked beam decreases as the crack depth increases and the crack location is constant.
2. Natural frequency of the cracked beam increases as the crack location increases from fixed end and the crack depth is constant.
3. The results of Finite Element Analysis and experimental analysis are compared and they are in good agreement.
4. The study showed small crack depth ratios had small effect on the sensitivity of the natural frequencies. It was also observed that the changes became more significant as the crack grew deeper.
5. The effect of crack is more pronounced when the cracks are near to the fixed end than at free end.
6. Experimental tests results can be used to detect and monitor fatigue cracks in beams, shafts or rotating machine element, from which the health of the element could be recorded at various stages of fatigue damage.
7. In the present study, the beams under consideration have uniform cross section but this method can be extended to components with varying cross section, different geometry and any boundary condition.
8. The proposed method can be extended for fault diagnosis in beams, shafts or rotating machine element.

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