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Fanno Flow and Rayleigh Flow Calculations for Bleed Flow through a Variable Area By-Pass Duct of a Pulse Detonation Engine

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Abstract

A pulse detonation engine, or "PDE", is a type of propulsion system that uses detonation waves to combust the fuel and oxidizer mixture. The flow after the inlet is divided into two sections: primary flow and secondary flow. The primary flow is the flow which goes through the main engine components i.e. the pulsed engine. The engine is pulsed because the mixture must be renewed in the combustion chamber between each detonation wave initiated by an ignition source. The secondary flow is the bypassed flow or the bleed flow which has a lower thrust specific fuel consumption. The study and analysis of bleed flow under different working conditions is the main objective of this project. The bleed flow will be studied under two cases: one involving frictional effects i.e. Fanno Flow and the other involving heat addition i.e. Rayleigh Flow. These two flows are constant area flows. Variable area flow will be studied with the help of the results derived from constant area flow.

Keywords: Fanno flow, Rayleigh Flow, Convective Heat Transfer Coefficient

1. Introduction

A single, axially-traveling detonation wave is initiated at the closed end of a tube partially filled with combustible mixture. The hot, high-pressure products then accelerate out of the device (sometimes through a nozzle). The remaining gases are purged and the process repeats in a cyclic manner (typically between 15-80 Hz)

PDEs are not self-aspirating, cannot self-ignite and are not steady-state machines. PDEs require complex, oscillatory subsystems (valves, plumbing, and ignition) to sustain operation. Dozens of government, industry and academic experimental programs have analysed PDEs to: – Explore the operational challenges of various liquid and gaseous propellants – Validate complex thermodynamic models – Anchor unsteady detonation combustion chemistry codes – Examine the effects of inlet and exhaust nozzle geometry on performance – Identify and address the challenges with downstream hardware integration (i.e. turbines) – Explore mitigation techniques for auto ignition.

2. Methodology

2.1 Rayleigh Flow

The general behaviour of an arbitrary fluid is considered initially. To isolate the effects of heat transfer the following assumptions are made

- Steady one-dimensional flow
- Negligible friction $ds_i \approx 0$
- No shaft work $\delta w_s = 0$
- Neglect potential $dz = 0$
- Constant area $dA = 0$

Rayleigh flow refers to adiabatic flow through a constant area duct where the effect of heat addition or rejection is considered. Compressibility effects often come into consideration, although the Rayleigh flow model certainly also applies to incompressible flow. For this model, the duct area remains constant and no mass is added within the duct. Therefore, unlike Fanno flow, the stagnation temperature is a variable. The heat addition causes a decrease in stagnation pressure, which is known as the Rayleigh effect and is critical in the design of combustion systems. Heat addition will cause both supersonic and subsonic Mach numbers to approach Mach 1, resulting in choked flow. Conversely, heat rejection decreases a subsonic Mach number and increases a supersonic Mach number along the duct. It can be shown that for calorically perfect flows the maximum entropy occurs at M = 1.

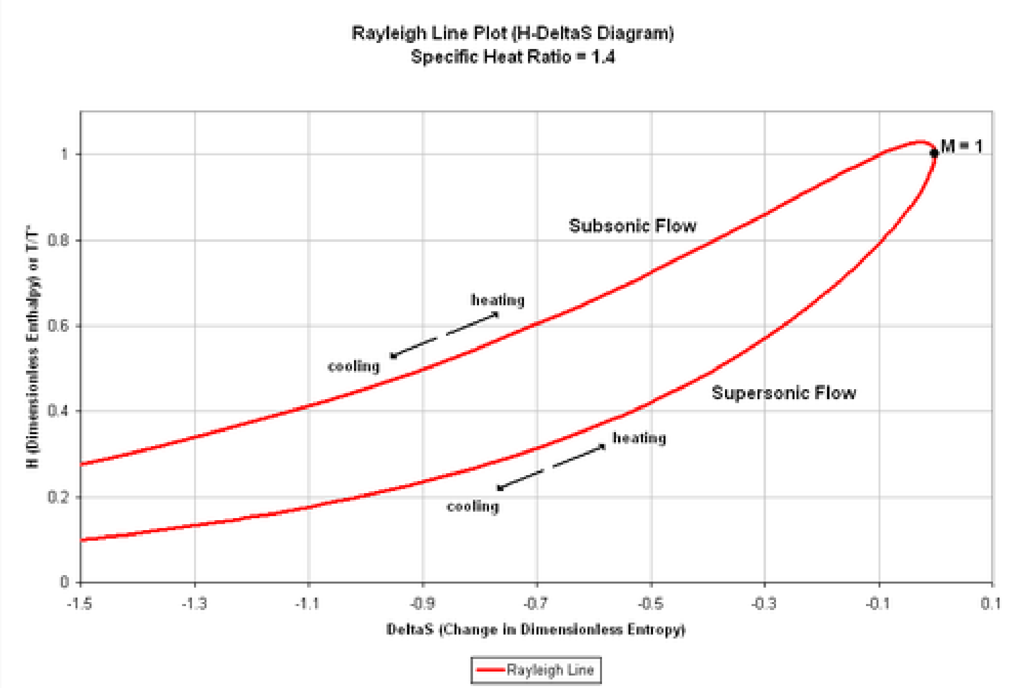

Figure 1 Rayleigh Line plotted on the dimensionless H-ΔS axis.

The Rayleigh flow model begins with a differential equation that relates the change in Mach number with the change in stagnation temperature, T_0. The differential equation is shown below.

$$\frac{dM^2}{M^2} = \frac{1+\gamma M^2}{1-M^2}\left(1+\frac{\gamma-1}{2}M^2\right)\frac{dT_0}{T_0}$$

Solving the differential equation leads to the relation shown below, where T_0* is the stagnation temperature at the throat location of the duct which is required for thermally choking the flow.

$$\frac{T_0}{T_0^*} = \frac{2\left(\gamma+1\right)M^2}{\left(1+\gamma M^2\right)^2}\left(1+\frac{\gamma-1}{2}M^2\right)$$

These values are significant in the design of combustion systems. For example, if a turbojet combustion chamber has a maximum temperature of T_0* = 2000 K, T_0 and M at the entrance to the combustion chamber must be selected so thermal choking does not occur, which will limit the mass flow rate of air into the engine and decrease thrust.

For the Rayleigh flow model, the dimensionless change in entropy relation is shown below.

$$\Delta S = \frac{\Delta s}{c_p} = ln\left[M^2\left(\frac{\gamma+1}{1+\gamma M^2}\right)^{\frac{\gamma+1}{\gamma}}\right]$$

The above equation can be used to plot the Rayleigh line on a Mach number versus ΔS graph, but the dimensionless enthalpy, H, versus ΔS diagram is more often used. The dimensionless enthalpy equation is shown below with an equation relating the static temperature with its value at the choke location for a calorically perfect gas where the heat capacity at constant pressure, c_p, remains constant.

$$H = \frac{h}{h^*} = \frac{c_pT}{c_pT^*} = \frac{T}{T^*}$$
$$\frac{T}{T^*} = \left[\frac{\left(\gamma+1\right)M}{1+\gamma M^2}\right]^2$$

The area and mass flow rate are held constant for Rayleigh flow. Unlike Fanno flow, the Fanning friction factor, *f*, remains constant. These relations are shown below with the * symbol representing the throat location where choking can occur.

$$A = A^* = \text{constant}$$
$$\dot{m} = \dot{m}^* = \text{constant}$$

Differential equations can also be developed and solved to describe Rayleigh flow property ratios with respect to the values at the choking location.

Working equation for static temperatures -

$$\frac{T_2}{T_1} = \left(\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right)^2 \frac{M_2^2}{M_1^2}$$

$$\frac{T_2}{T_1} = \frac{p_2^2 M_2^2}{p_1^2 M_1^2}$$

2.2 Fanno Flow

To study only the effect of friction, the Fanno Flow analysis in the duct is dealt with.
The general behavior of an arbitrary fluid is considered initially. To isolate the effects of friction, the following assumptions are made:

- Steady one-dimensional flow
- Adiabatic $\delta q = 0$, $ds_e = 0$
- No shaft work $\delta w_s = 0$
- Neglect potential $dz = 0$
- Constant area $dA = 0$

By applying the basic concepts of continuity, energy, and momentum;

Continuity

$$m = \rho AV = \text{const}$$

But since the flow area is constant, this reduces to

$$\rho V = \text{const}$$

A new symbol G is assigned to this constant (the quantity ρV), which is referred to as the mass velocity, and thus

$$\boxed{\rho V = G = \text{const}}$$

Energy

$$h_{t1} + q = h_{t2} + w_s$$

For adiabatic and no work, this becomes

$$h_{t1} = h_{t2} = h_t = const$$

Neglecting the potential term, this means that

$$h_t = h + V^2/2g_c = const$$

Thus,

$$h_t = h + G^2/\rho^2 2g_c = const$$

Which will be equal to

$$h_t = h + V^2/2g_c = constant$$

Differentiating,

$$dh_t = dh + VdV/g_c = 0$$

From continuity

$$\rho V = G = constant$$

Differentiating this,

$$\rho\, dV + V\, d\rho = 0$$

Which can be solved for

$$dV = -Vd\rho/\rho$$

Thus

$$dh = V^2 d\rho/g_c\rho$$

Now recalling the property relation

$$T\, ds = dh - v\, dp$$

Which can be written as

$$T\, ds = dh - dp/\rho$$

Substituting for dh yields

$$\boxed{T\, ds = V^2 d\rho/g_c\rho - dp/\rho}$$

Momentum

The foregoing analysis was made using only the continuity and energy relations. Now proceeding to apply momentum concepts to the control volume shown in Figure 1.

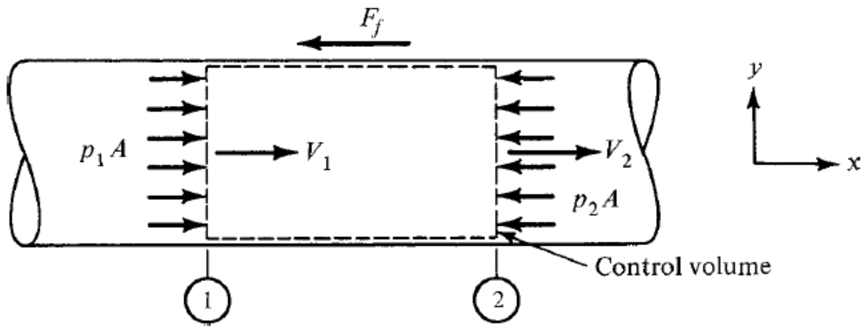

Figure 2 Control Volume

The x-component of the momentum equation for steady, one-dimensional flow is

$$\sum F_x = mg_c(V_{outx} - V_{inx})/g_c$$

From Figure 1 we see that the force summation is

$$\sum F_x = p_1A - p_2A - F_f$$

Where F_f represents the total wall frictional force on the fluid between sections 1 and 2. Thus the momentum equation in the direction of flow becomes

$$(p_1 - p_2)A - F_f = m\,(V_2 - V_1)/\,g_c = \rho AV\,(V_2 - V_1)/\,g_c$$

Which can be written as

$$p_1 - p_2 - F_f/A = \rho_2 V_2^2/g_c - \rho_1 V_1^2/g_c$$

Or

$$p_1 + \rho_1 V_1^2/g_c - F_f/A = p_2 + \rho_2 V_2^2/g_c$$

In this form the equation is not particularly useful except to bring out one significant fact. For the steady, one-dimensional, constant-area flow of any fluid, the value of p+ρV2/g cannot be constant if frictional forces are present.

In the differential form this can be written as

$$dp/\rho + fV^2dx/2g_cD + gdz/g_c + dV^2/2g_c = 0$$

The objective is to get this equation all in terms of Mach number. If we introduce the perfect gas equation of state together with expressions for Mach number and sonic velocity, we obtain

$$dp/p\,(RT) + fdxM^2\gamma RT/2D + gdz/g_c + (dM^2\gamma g_cRT + M^2\gamma g_cRdT)/2g_c = 0$$

Or

$$\boxed{dp/p + fdx\gamma M^2/2D + gdz/g_cRT + \gamma dM^2/2 + \gamma M^2dT/2T = 0}$$

This equation is important since it is a useful form of the momentum equation that is valid for all steady flow problems involving a perfect gas.

This equation deals with the constant area Fanno Flow.

By applying the method of influence coefficients to this equation (Curie 1975, Zucrow 1976), the governing differential equation becomes

$$dA/Adx + [(1-M^2/M)(dM/dx)]/[1+(\gamma-1)M^2/2] = \gamma M^2 f/2D$$

The pressure can be evaluated by the differential equation

$$dp/Pdx = \gamma M^2 dA/Adx(1-M^2) - \gamma M^2[1+(\gamma-1)M^2]f/2D(1-M^2)$$

2.3 Convective Heat Transfer Coefficient (h_c)

To calculate the Heat flux generated at a given temperature the convective heat transfer coefficient has to be calculated first,

The basic concepts of Heat Transfer will be used which include Nusselt's number, Graschoff's number and Prandlt's number which will be further used to calculate the Convective Heat Transfer Coefficient.

As we know,

$$h_c = (N_u * k)/L$$

where,

for horizontal turbulent flow,

$$N_u = 0.14 * R_a^{0.33}$$

$$R_a = G_r * P_r$$

and,

$$G_r = (g * L^3 * \beta(T_p - T_a))/\eta^2$$

$$T_p = 450K, T_a = 320K, g = 9.81m/s^2, L = 1.2m, \eta = 1.73 * 10^{-5} m^2/s$$

Thus the Grashof's number is calculated to be = 2.30 * 10^{10} = G_r

As we know,

$$P_r = \mu C_p/k$$

$$\mu = 1.94 * 10^{-5}$$

Thus Prandlt's Number is calculated to be = 0.75 =P_r

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Since we know both, the Prandlt and the Grashof number, we can calculate R_a.

$$R_a = G_r * P_r$$

Thus the value of R_a = $2.213*10^9$

Hence Nusselt number can be calculated by the equation -

$$N_u = 0.14 * R_a^{0.33}$$

Thus the value of Nusselt number is calculated to be = 363.03 = N_u

Hence the Convective Heat Transfer Coefficient can be calculated by the equation

$$h_c = (N_u * k)/L$$

Thus the value of Convective Heat Transfer Coefficient is calculated to be h_c = 7.86 W/m^2K

3. Calculations

The pressure variations will be calculated for the Bypass Area which is highlighted by the bold lines. The dimensions are shown in the figure.

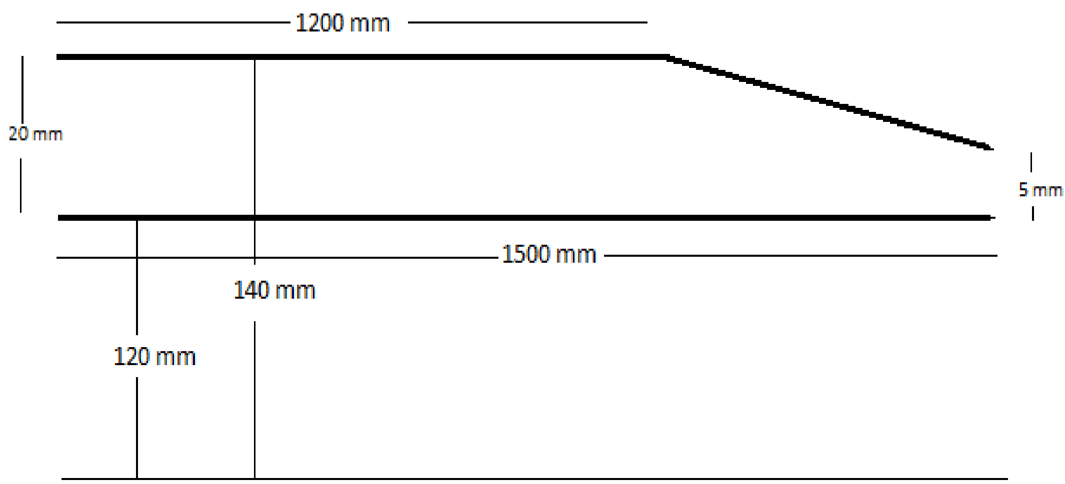

Figure 3 Sketch of the side view of bypass section

3.1 Fanno Flow

Boundary Conditions

Inlet Pressure = 1.6 bar

Inlet Temperature = 320 K

The friction factor 'f' is taken from the Moody diagram, as per the Reynold's number calculation.

$Re = \rho ud/\mu = 170000$

The value of Reynold's number suggests that the flow is turbulent.

The friction factor thus is 0.02

Case 1 – Mach = 0.2

$u_1 = M_1 \times a = 67$ m/sec

$P_2 = 1.591$ bar

$P_3 = 1.463$ bar

Case 2 – Mach = 0.4

This is a case of compressible flow. Thus, the density variation will be taken into account

$\rho = P/RT = 160000/ (287 \times 320) = 1.742$ kg/m^3

$u_1 = M_1 \times a = 134.16$ m/sec

$P_2 = 1.54$ bar

$P_3 = 1.048$ bar

3.2 Rayleigh Flow

Now for calculating the pressure losses for the case of Rayleigh Flow, in which the pressure drop is caused by the heat flux through the walls, the converging section of the bypass area is divided into six different sections of equal lengths as shown in the following figure

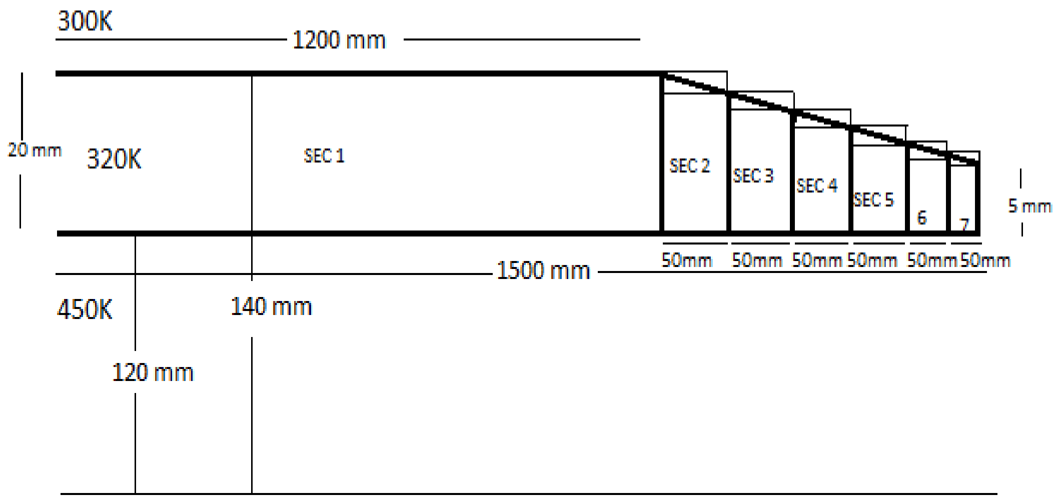

Figure 4 Area calculation by section method

As can be seen from the figure the entire test section has been divided into 7 sections. Now the areas of all the sections will be calculated individually.

T_1=320K, P_1=1.6 bar, γ=1.4, M=0.2

The area of section 1 is calculated by using the following formula

$A_1 = \pi l d_1$= 0.90432m^2 (d_1=0.24m)

A_2= $\pi l d_2$= 1.05504m^2 (d_2=0.28m)

Now the heat flux is calculated by the following formula-

q_1=$h_c A_1 \Delta T$ = 924.034W ($\Delta T = T_2$-T) (T_2=450K,T=320K)

q_2=$h_c A_2 \Delta T$ = 165.852W ($\Delta T = T - T_1$) (T=320K,T_1=300K)

Net heat flux-

q = q_1 - q_2 = 758.182 W

Now to calculating the temperature (T) for section 2

q = $C_p \Delta T$ = C_p(x-320)

x = T = 320.750K (Temperature at section 2) = T_2

Now the area of the converging section is to be calculated. To calculate the area of the converging, the converging section is divided into 5 sections of equal lengths and calculate the area of each section individually.

Calculating the Area of Section 2 -

$A_1=\pi d_1$*L = 0.03768m^2 (d_1=0.24m, L=0.05m)

$A_{21}=\pi d_{11}$*L = 0.04396 m^2 (d_{11}=0.28m, L=0.05m)

$A_{22}=\pi d_{12}$*L = 0.04356 m^2 (d_{12}=0.2775m, L=0.05m)

A_2 = (A_{21}+ A_{22})/2 = 0.043763 m^2

Calculating the Heat Flux at section 2 -

$q_1=h_c A_1 \Delta T$ = 38.279W (h_c=7.86, T_2=450K, T=320.750)

$q_2=h_c A_2 \Delta T$ = 7.137W (T_1=300K, T=320.750K)

q = q_1 - q_2 = 31.142W

Now calculating the temperature (T) for section 3 -

q = $C_p \Delta T$ = C_p (x-320.750)

x = 320.780K (Temperature at section 3)

Now calculating the area of section 3 -

$A_1=\pi d_1 * L$ = 0.03768m^2

$A_{21}=\pi d_{11} * L$ = 0.0435675m^2

$A_{22}=\pi d_{12} * L$ = 0.0427825m^2

A_2= (A_{21}+A_{22})/2 = 0.0429787m^2

Now calculating the Heat Flux at section 3 -

q_1 = $h_c A_1 \Delta T$ = 38.270 W (T_2=450, T=320.780)

q_2 = $h_c A_2 \Delta T$ = 7.029 W (T_1=300, T=320.780)

q = q_1 - q_2 = 31.231W

Now calculating the temperature (T) at section 4 -

q = $C_p \Delta T$ = C_p(x - 320.780)

x = 320.810K

Now calculating the area of section 4 -

A_1=$\pi d_1 * L$ = 0.03768m^2

A_{21}= $\pi d_{11} * L$ = 0.0431750m^2

A_{22}= $\pi d_{12} * L$ = 0.0427825 m^2

A_2= $(A_{21} + A_{22})$/2 = 0.0429787m^2

Now calculating the Heat Flux for section 4 -

q_1=$h_c A_1 \Delta T$ = 38.261W (T_2=450K, T=320.810K)

q_2=$h_c A_2 \Delta T$ = 7.029W $(T_1 = 300K, T = 320.810K)$

q = $q_1 - q_2$ = 31.231W

Now calculating the temperature (T) at section 5 -

q = $C_p \Delta T$ = $C_p(x - 320.810)$

x = 320.840K

Now again calculating the area, now for section 5 -

A_1=$\pi d_1 * L$ = 0.03768m^2

A_{21}=$\pi d_{11} * L$ = 0.0427825m^2

A_{22}=$\pi d_{12} * L$ = 0.042390m^2

A_2= $(A_{21} + A_{22})$/2 = 0.042586m^2

Now calculating the Heat Flux at section 5 -

q_1=$h_c A_1 \Delta T$ = 38.255W $(T_2 = 450K, T = 320.840K)$

q_2=$h_c A_2 \Delta T$ = 6.971W $(T_1 = 300K, T = 320.840K)$

q = $q_1 - q_2$ = 31.282W

Now calculating the temperature (T) at section 6 -

q = $C_p \Delta T$ = $C_p\ (x - 320.840)$

x = 320.870K

Calculating the area of section 6 -

A_1=$\pi d_1 * L$ = 0.03768m^2

A_{21}=$\pi d_{11} * L$ = 0.042390m^2

A_{22}=$\pi d_{12} * L$ = 0.041997m^2

A_2= $(A_{21} + A_{22})/2$ = 0.042193m^2

Now calculating the Heat Flux at section 6 -

q_1=$h_c A_1 \Delta T$ = 38.243W

q_2=$h_c A_2 \Delta T = 6.921$W

q = q_1 - q_2 = 31.321W

Now calculating the temperature (T) at section 7 -

q = $C_p \Delta T$ = $C_p(x - 320.870)$

x = 320.900K

Calculating the area of section 7 -

A_1=$\pi d_1 * L$ = 0.03768m^2

A_{21}=$\pi d_{11} * L = 0.0419297m^2$

A_{22}=$\pi d_{12} * L = 0.041605m^2$

A_2= $(A_{21} + A_{22})/2$ = 0.041801m^2

Calculating the Heat Flux at section 7 -

q_1=$C_p \Delta T$ = 38.41W

q_2=$C_p \Delta T$ = 6.86W

q = $q_1 - q_2$ = 31.543W

Now calculating the Final Temperature of the air leaving the test section -

q = $C_p \Delta T$ = $C_p\,(x - 320.900)$

x = 320.930K = T_3

Now the entire test section will be divided into three major sections, namely sections 1, 2 and 3 respectively as shown in the figure

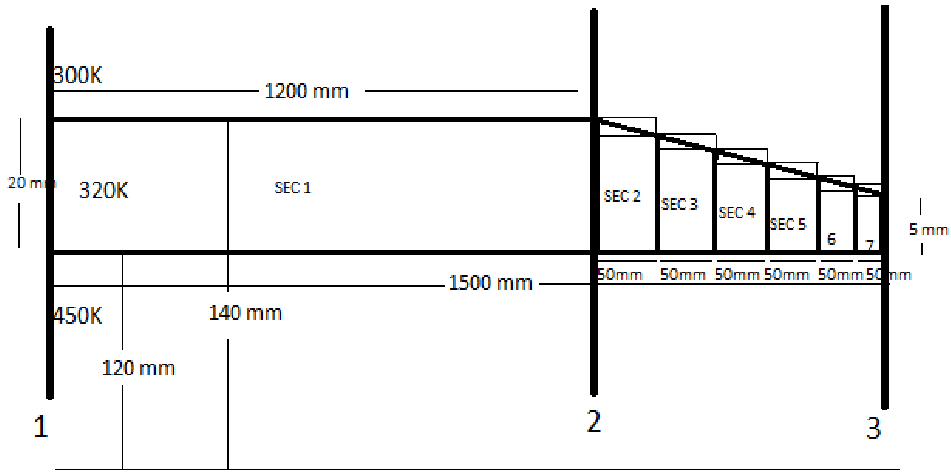

Figure 5 Division in different sections

Now to calculate the pressure drop at section 2, Mach no. at section 2 will be calculated.

As, $T_1 = 320K, T_2$ = 320.750K, $M_1 = 0.2$, $M_2 = ?$, $\gamma = 1.4$

By using the formula -

$$\frac{T_2}{T_1} = \left(\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right)^2 \frac{M_2^2}{M_1^2}$$

The Mach no. at section 2 is calculated as 0.201 = M_2.

Now Pressure at section 2 is calculated by using the equation -

$$\frac{T_2}{T_1} = \frac{p_2^2 M_2^2}{p_1^2 M_1^2}$$

By using this equation the pressure comes out to be 1.593 bar = P_2.

The Mach number and Pressure at section 3 can be calculated in the same manner.

Mach number at section 3 (M_3) -

T_2 = 320.750K, $T_3 = 320.930K$, M_2 = 0.201, M_3 = ?

Using the equation -

$$\frac{T_2}{T_1} = \left(\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right)^2 \frac{M_2^2}{M_1^2}$$

The Mach number at section 3 is calculated as 0.209 = M_3.

Pressure at section 3 (P_3) −

P_2=1.593 bar, T_2 = 320.750K, $T_3 = 320.930K$, M_2 = 0.201, M_3 = 0.209, P_3=?

Using the equation -

$$\frac{T_2}{T_1} = \frac{p_2^2 M_2^2}{p_1^2 M_1^2}$$

The Pressure at section 3 is calculated to be 1.5321 bar = P_3 = Final Pressure at Outlet of the test section.

Sections	Inlet Temperature	Outlet Temperature	Net Heat flux (W)	Net Area(m^2)
Section 1	320K	320.750K	758.182	0.15072
Section 2	320.750K	320.780K	31.142	0.0058875
Section 3	320.780K	320.810K	31.187	0.0056912
Section 4	320.810K	320.840K	31.231	0.0052987
Section 5	320.840K	320.870K	31.282	0.004906
Section 6	320.870K	320.900K	31.321	0.004513
Section 7	320.900K	320.930K	31.543	0.004121

Table 1 Calculated data of Rayleigh flow analysis

4. Conclusions

The analysis of bleed flow of a pulse detonation engine was studied separately for Fanno and Rayleigh flows. The following conclusions were drawn from the analysis:

- Pressure drops along the length as the flow passes from the duct.
- At the constant area section, the pressure drop is less than the pressure drop at variable area section
- Pressure drop in the Fanno flow is more as compared to the Rayleigh flow
- Pressure drop in the compressible flow (Mach 0.4) is more as compared to incompressible flow (Mach 0.2)

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