INTERNATIONAL JOURNAL OF RESEARCH IN AERONAUTICAL AND MECHANICAL ENGINEERING

ISSN (ONLINE): 2321-3051

Vol.3 Issue.5,

May 2015.

Pgs: 10-17

Numerical Scheme for 2-Dimension Optimum Length Nozzle Contour design imposing Viscous Correction to Method of Characteristics

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Abstract

A new numerical method is proposed to design optimum length nozzle contours imposing viscous correction to Method of Characteristics. Proposed method utilizes the information from the characteristic lines to design the contour which has wide applications in the field of Aerospace and Aeronautics. Nozzle contours for both circular and rectangular cross sections are developed and various numerical experimentations were performed to analyze the performance of the scheme proposed. A comparative study on optimum number of characteristics required for optimum design is presented. 3D contour design will be carried out as future research.

***Keywords**:* Optimum length nozzle, Method of characteristics, Contour design, 2D nozzles, Characteristic lines.

1. Introduction

Method of characteristics forms the major branch in the numerical techniques used to analyze steady supersonic flows. The keystone of theoretical fluid dynamics is a set of conservation equations that describe the physics of fluid motion; These set of conservation equation states that Mass is conserved (Continuity Equation), Momentum is conserved (Navier stokes Equation), Energy is conserved (Energy Equation). They describe the variation in the fluid pressure, temperature, density, velocity throughout the entire flow field both in space and time. Since the time of formulation of these general conservation equations, attempt to find an analytical solution were begun. Though analytical solutions were formulated for special cases of flows like flow over thin airfoil, flow over a flat plate, flow over round corner etc., a general solution to the entire equation was never formulated.

The method of characteristics is an older and more developed, and is limited to inviscid flows, whereas finite-difference techniques are still evolving as computational fluid dynamics grows and matures, and have much more general application to inviscid and viscous flows. So, a new numerical scheme was developed for Supersonic nozzle contour design using method of characteristics was developed.

2. Formulation of method of characteristics

Though the method of characteristics is limited to Inviscid flows, Continuity and Momentum equations are alone required to predict the parameters in the grid points. By combining both these equations, the velocity potential equation is given as,

$$\left(1-\frac{\Phi_x^2}{a^2}\right)\Phi_{xx}+\left(1-\frac{\Phi_y^2}{a^2}\right)\Phi_{yy}+\left(1-\frac{\Phi_z^2}{a^2}\right)\Phi_{zz}$$

$$-\frac{2\Phi_x\Phi_y}{a^2}\Phi_{xy}-\frac{2\Phi_x\Phi_z}{a^2}\Phi_{xz}-\frac{2\Phi_y\Phi_z}{a^2}\Phi_{yz}=0$$

Flow field properties for any grid shown below are determined based on their ability to be expanded using Taylor series as follows,

$$u_{i+1,j}=u_{i,j}+\left(\frac{\partial u}{\partial x}\right)_{i,j}\Delta x+\left(\frac{\partial^2 u}{\partial x^2}\right)_{i,j}\frac{(\Delta x)^2}{2}+\cdots$$

Neglecting the higher order terms, the expansion series can be written as,

$$u_{i+1,j}=u_{i,j}+\left(\frac{\partial u}{\partial x}\right)_{i,j}\Delta x$$

The value of the derivative can be obtained from the general conservation equations. For example, consider a two-dimensional Irrotational flow, so that the velocity potential equation yields, in terms of velocities,

$$\left(1-\frac{u^2}{a^2}\right)\frac{\partial u}{\partial x}+\left(1-\frac{v^2}{a^2}\right)\frac{\partial v}{\partial y}-\frac{2uv}{a^2}\left(\frac{\partial u}{\partial y}\right)=0$$

Solving for $\partial u/\partial x$ gives:

$$\frac{\partial u}{\partial x}=\frac{\left[\frac{2uv}{a^2}\left(\frac{\partial u}{\partial y}\right)-\left(1-\frac{v^2}{a^2}\right)\frac{\partial v}{\partial y}\right]}{\left(1-\frac{u^2}{a^2}\right)}$$

Now, assume that we know the velocity V and its components (u,v) along a vertical line (say inlet). Thus we know the value of $u_{i,j}$ By finding the gradient in the x-direction from the above equation the value of $u_{i+1,j}$ can be found using Taylor series expansion about the point (i,j). However there is one notable exception, if the denominator in the above equations tends to zero, the derivative does not exist. The determinant is zero when u=a, i.e., when the component of flow velocity perpendicular to inlet is sonic. The line that makes a Mach angle with respect to the streamline direction at a point is also a line along which the derivative of u is indeterminate, and across which it may be discontinuous. These lines exist, and they are called the CHARACTERISTIC LINES or Mach Lines.

Let us move towards the supersonic 2d inviscid Irrotational flow in the divergent section of the nozzle. The problem can be solved in 3 steps.

2.1 Algorithm:

1. Find some particular lines (directions) in the x y space where flow variables (p, T, u, v, ρ) are continuous, but along which the derivatives ($\partial u/\partial x$, $\partial p/\partial x$ *etc*.) are indeterminate, and in fact across which the derivatives may even sometimes be discontinuous. As already defined, such lines in the xy space are called characteristic lines.

2. Combine the partial differential conservation equations in such a fashion that ordinary differential equations are obtained that hold only along the characteristic lines. Such ordinary differential equations are called the compatibility equations.
3. Solve the compatibility equations step by step along the characteristic lines, starting from the given initial conditions at some point or region in the flow. In this manner, the complete Flow field can be mapped out along the characteristics. In general, the characteristic lines depend on the Flow field, and the compatibility equations are a function of geometric location along the characteristic lines' hence, the characteristics and the compatibility equations become algebraic equations explicitly independent of geometric location.

2.2 Determination of characteristic lines

Consider steady, adiabatic, two-dimensional, Irrotational supersonic flow. The governing non-linear equations are as follows,

$$\left(1-\frac{\Phi_x^2}{a^2}\right)\Phi_{xx}+\left(1-\frac{\Phi_y^2}{a^2}\right)\Phi_{yy}-\frac{2\Phi_x\Phi_y}{a^2}\Phi_{xy}=0$$

$$d\Phi_x=\frac{\partial\Phi_x}{\partial x}dx+\frac{\partial\Phi_y}{\partial y}dy=\Phi_{xx}\,dx+\Phi_{xy}\,dy$$

$$d\Phi_y=\frac{\partial\Phi_y}{\partial x}dx+\frac{\partial\Phi_y}{\partial y}dy=\Phi_{xy}\,dx+\Phi_{yy}\,dy$$

Where, $\Phi_x=u\,;\ \Phi_y=v$

This provides us a means to calculate the equations of the characteristic lines.

$$\begin{vmatrix} 1-\frac{u^2}{a^2} & -\frac{2uv}{a^2} & 1-\frac{v^2}{a^2} \\ dx & dy & 0 \\ 0 & dx & dy \end{vmatrix}=0$$

The above system yields the slope of the characteristic line as follows since it is a quadratic equation in slope of the characteristic line.

$$\left(\frac{dy}{dx}\right)_{char}=\frac{-\frac{uv}{a^2}\pm\sqrt{M^2-1}}{\left(1-\frac{u^2}{a^2}\right)}$$

Tthe supersonic flows are hyperbolic in nature and two characteristics runs at every point in the flow, this makes the method of characteristics a practical technique for solving supersonic flows. In contrast, since the characteristics are imaginary the method of characteristics is not used for subsonic solutions.

Since the contour design problem for a supersonic nozzle is a 2 dimensional, steady, supersonic flow problem. Method of characteristics is the tranquil way of determining the divergent portion of supersonic nozzle. At any point u = V cos(θ), v = V sin(θ). Hence the slope of the characteristic line can be said to be equal to

$$\left(\frac{dy}{dx}\right)_{char} = \tan(\theta \mp \mu)$$

2.3 Compatibility equation

$$d\theta = \mp\sqrt{M^2 - 1}\frac{dV}{V}$$

This equation is called as the compatibility equation. Two values of the $d\theta$ are obtained among which the negative one corresponds to the negative characteristic line or left running characteristic and so on. The above equation is similar to Prandtl Meyer expansion flow equation, so they can be integrated to give the Prandtl Meyer function $\nu(M)$. Therefore they can be replaced by algebraic compatibility equation.

$\theta + \nu(M) = const = K_-\ (along\ the\ C_-\ characteristic)$

$\theta - \nu(M) = const = K_+\ (along\ the\ C_+\ characteristic)$

The above equations relate the velocity magnitude and the direction along the characteristic lines. For this reason, they are sometimes identified in the literature as 'hodograph characteristics'. Since the compatibility equations are independent of the spatial coordinates x and y. Hence, they can be solved without requiring knowledge of the geometric location of the characteristic lines. This is peculiar case applicable only to two-dimensional Irrotational flow.

3 Viscous correction to inviscid method of characteristics

Since the Method of Characteristics is only applicable for Inviscid flows, a viscous correction is required to account for Viscous induced Effects.

Due to the viscous effects, the effective turn angle at the wall is reduced, thereby reducing the expansion angle, where a viscous correction angle is introduced as follows,

$$\theta_{viscous} \pm \nu(M) = const$$

$$\theta_{viscous} = \theta - \delta\left(\left(\frac{\mu}{\rho VL}\right), x\right)$$

Where,

δ – Correction function for viscous effects

μ – Dynamic Viscosity

Now assuming Laminar flow inside nozzle, and boundary layer growth and other viscous related effects occur only in the wall of nozzle, correction function can be approximated to displacement boundary layer thickness. Thus the Viscous correction is made only in the wall grid points. Final expression for viscous correction function can be stated as (from Blasius Solution for Flat Plates).

$$\delta(Re,x) = \frac{5x}{\sqrt{\frac{\mu}{\rho VL}}}$$

This assumption leads to an approximation, that consecutive grid points are assumed to be joined by linear lines (flat plates) and the distance x varies from 0 to Δx, where Δx is the grid size.

Finally the Viscous Corrected Method of Characteristics Scheme can be summarized as follows

On wall Grid points	$\theta_{viscous} \pm \nu(M) = const$
On internal Grid points	$\theta \pm \nu(M) = const$

4 Numerical experiments

Proposed Scheme is tested for various inlet conditions at throat of nozzle, and for various number of characteristic line designs. One of the results for a specific Test condition is presented in this paper. 10, 20 and 100 characteristic designs for the Throat Inlet condition given below are presented.

Throat Inlet Condition

Temperature	: 1200 K
Pressure	: 1.2E6 bar
Ambient Pressure	: 1.01325E5 bar
Ambient Temperature	: 288.16 K
Specific Ratio	: 1.4
Molecular Mass of gas	: 25.4
Number of Characteristics	: Chosen (10,20,100)
Initial Theta	: 0.03 radian
Area of the Throat	: 0.1 m^2

5 Results and discussions

Figure 1, shows the 2D contour design using viscous corrected scheme of Method of Characteristics. Approximately all the three designs produce contour with exit area 0.58 sq. m. Thus the scheme achieves grid independence for even less number of characteristic lines (i.e. very less number of characteristics lines are adequate to achieve high accurate contour design).

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Vol.3 Issue.5, May 2015. Pgs: 10-17

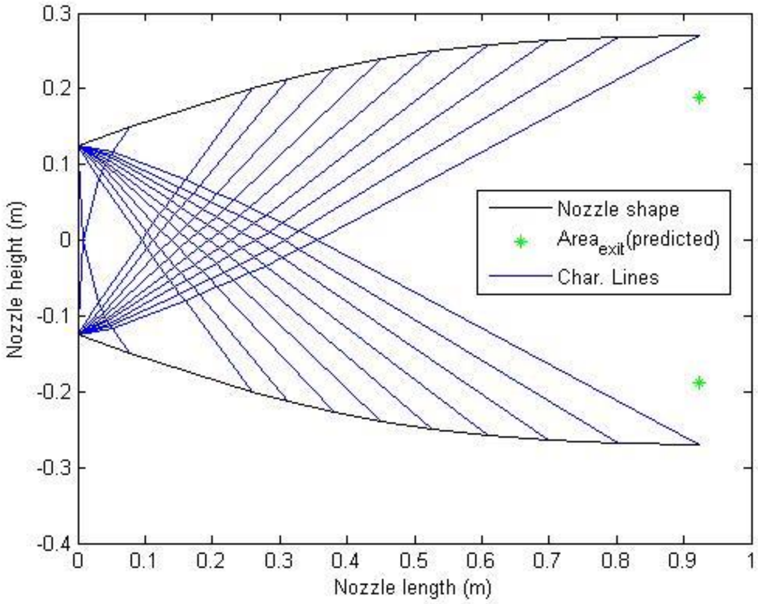

Figure 1: Contour design using 10 Characteristic lines

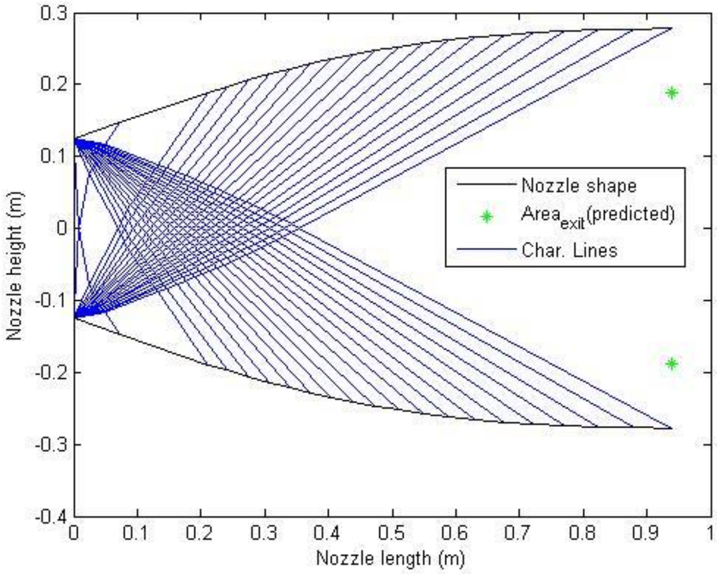

Figure 2: Contour design using 20 Characteristic lines

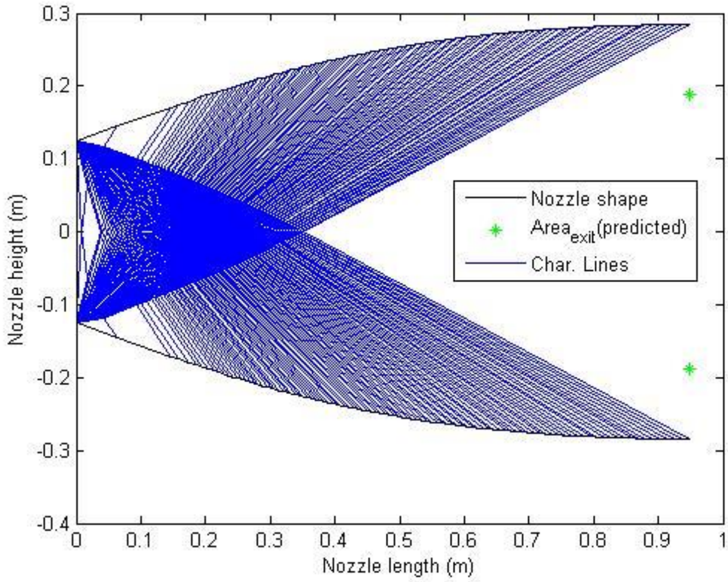

Figure 3: Contour design using 100 Characteristic lines

6 Conclusion

Viscous corrected method of characteristics scheme creates nozzle contour where there isn't much deviation for increased number of characteristic lines. Solutions using 10, 20 & 100 characteristic lines were presented shows not much deviation. Viscous corrected scheme accounts for the viscous boundary layer effects inside the diverging region of the nozzles. Proposed scheme performs better than most existing schemes used for 2D Nozzle contour design.

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