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COMPARITIVE STUDY OF BAYESIAN APPROACH AND LEAST SQUARE RESIDUAL OPTIMIZATION METHOD IN HORIZONTAL HEATED PLATE FACING UPWARD-AN EXPERIMENTAL APPROACH

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Abstract

Free convection heat transfer is encountered in many engineering applications such as Cooling of electronic equipment, pollution, materials processing, energy systems, and safety in thermal processes and geophysical flows. The present work discusses about the result of an experimental study of Steady, Laminar, Free convection heat transfer in a horizontal plate facing upwards in which two walls are adiabatic and other two ends are open to the ambient .The aim is to estimate the value of constant C in the Nusselt number of available correlation using least square residual method and Bayesian approach where air is used as a working medium. The steady temperature time history is mainly used to estimate constant of the Nusselt number in both optimization method. The experimental setup has been designed and fabricated. Finally, Estimated Parameter is compared with Experimental benchmark.

***Keywords* :** Free Convection, Least Square Residual Method, Bayesian Approach

1. Introduction

Natural Convection is mainly due to the density differences between the solid and fluid surface (T_s>T_a). The density difference gives rise to buoyancy forces due to which the flow is generated. A heated body cooling in ambient air generates such a flow in the region surrounding it. (T_s>T_a). The buoyant flow arising from heat or material rejection to the atmosphere, heating and cooling of rooms and buildings, recirculating flow driven by temperature and salinity differences in oceans, and flows generated by fires are other examples of natural convection.There has been growing interest in buoyancy-induced flows and the associated heat and mass transfer over the past several decades, because of the importance of these flows in many different areas, such as cooling of electronic equipment, pollution, materials processing, energy systems, and safety in thermal processes.

(Elenbass,1942) conducted experimental work in laminar natural convection heat transfer in smooth parallel plate vertical channel was investigated and reported a detailed study of the thermal characteristics of cooling by natural convection.(R.A. Wirtz et al.,1982) have considered a geometry with constant heat sources placed over the entire length of the wall. Since the geometry cannot simulate discrete placement of chips, a number of discrete heat sources placed over a wall was considered by, (Chen Linhui et al., 2006).(Osterle,1962) conducted numerical analysis on free convection heat transfer for development of boundary layer between parallel isothermal vertical plates and get result for velocity, temperature and pressure variation throughout the flow field .The numerical method used is hybrid finite difference method.(Yousef et al.,1982) conducted an experimentally study of free convection heat transfer in air from isothermal horizontal surfaces heated and facing upward by using a Mach-Zehnder interferometer. The local and the average heat-transfer coefficients and the temperature distributions were determined and compared with available experimental and theoretical results.. The nature of the free convection flow over the heated surface and the separation of the boundary layer were inferred from these random changes in the local and average Nusselt numbers.(Oztop et al., 2004) conducted numerical investigation of natural convection heat transfer in a square cavity with a heated plate placed in vertical and horizontal manner. The governing equations were solved with TDMA using finite difference equation based on the finite control volume approach with non- staggered grid arrangement and SIMPLEM algorithm. Computation was done in Rayleigh number ranging from10^4 to 10^6 at different aspect ratios and position of heated plate. Air was used as a working fluid (Pr=0.71).The effect of the position and aspect ratio of heated (Vertical and horizontal) plate on heat transfer and flow in square cavity were investigated. The result showed that as Rayleigh number increases with increase in mean Nusselt number at both vertical and horizontal locations position. At higher Ra numbers, when the plate is located horizontally heat transfer is decreased about 80% less than for vertical located position.(Krishnan and Balaji,2004) conducted a synergistic approach to parameter estimation in multimode heat transfer. This paper reports the efficacy of the least square residual method in parameter estimation when more than one mode of heat transfer is encountered. (Parthasarathy and Balaji, 2008) determined multiple parameter estimation in 2d conduction using Bayesian inference. (Venugopal and Balaji, 2008) conducted experimental analysis of transient heat transfer to recall the constant c in the Nusselt number using hybrid optimization techniques.

From literature survey it can be inferred that free convection in vertical channel geometry with discrete heat source and optimization techniques used in parameter estimation has considerable attention. This geometrical configuration has physical relevance with respect to electronic chip placement. The goal of the study is to compare the value of Nusselt number constant C with standard correlation of vertical plate using least square residual method and Bayesian approach.

2. Experimental Methodology

As a first step, a validation experiments has been conducted by studying the heat transfer in an isolated heated horizontal plate. The details of the experiment setup is shown in Table 1

Table 1 Details of Experimental Set-up

SI NO	Description	Dimension/Range
1	Rectangular Horizontal SS Plate (1 no.)	250×50× 3mm
2	Rectangular Box with Thermocol (2 no.)	500 x 500x 150 mm

3	Heat Input	0-300 W
4	Single Phase Closed Type Dimmerstat	0-1 Amps
5	Ammeter	0-1 Amps
6	Multimeter	350 V_{AC}
7	K-type Thermocouple with indicator	0-1000° C

Experimental apparatus has been specially planned and formulated to carry out investigations on electronics devices. The experimental setup consist of an apparatus, temperature indicator, K –type thermocouple, Multimeter, Ammeter, and AC power supply whose schematic diagram is as shown in Figure1.

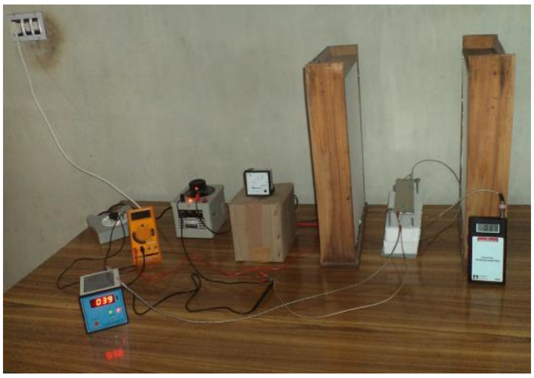

Figure 1 Photographic View of the Experimental setup

The apparatus consists of a stainless steel plate and flat heater fitted on a support structure in a horizontal fashion. The heat input to the heater is measured by an ammeter and a multimeter and is varied by a dimmerstat.Ammeter are connected in series manner and multimeter are connected in Parallel manner. The two rectangular box are kept as adiabatic walls which is filled with thermocol to avoid the external disturbance on both sides and only heat is supplied to the horizontal plate Temperatures is measured using K-type thermocouple. Thermocouples is fixed in between the flat heater and stainless steel plate with the help of blind holes by using the nuts and bolt on the support structure and taken out of the plate. The K type thermocouples are connected to the temperature indicator to measure the temperature of the plate. The stainless steel plate has been polished to minimize the radiation losses. As it could be seen there are 2 holes on the plate which house the screws that are used in fastening the plate and the heater together. The parameter varied during the experimentation is heat input to the heater. The whole assembly was highly polished on its outer surface to obtain an emissivity of 0.17

2.1 Procedure for the least Square Residual Optimization Method and optimization method

The experimentation consists of plate heater assembly which was given definite amount of power supply that was switched off after steady state was reached. Temperature measurements were done at regular interval of five minute. The experiment was carried out under controlled condition; simulating natural convection .

In this method, typically all the properties of the system will be known beforehand or a priori and the temperature response of the system will be sought. However, in this case the experimentally measured temperature response is available and to estimate or retrieve the value of C.

3. Results and Discussion

3.1 Least Square Residual Method

The Yousef et al correlation for Horizontal plate facing upward losing heat by free convection is

$$Nu = C\,Ra^{n} = 0.62Ra^{0.25}$$

The constant C in the above relation was determined by least square residual method and Bayesian Approach.

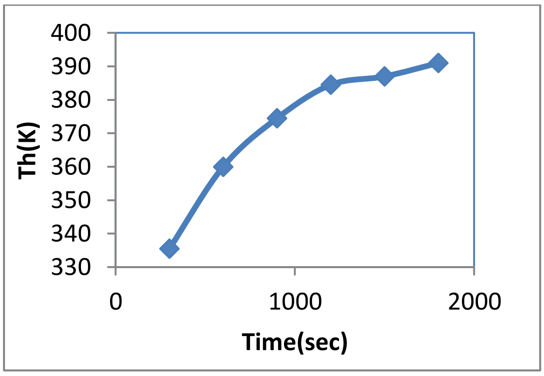

Figure 2 Plate temperature Vs Time

Figure 2 shows the Variation of Plate temperature with Time. Time increases with increase in plate temperature due to the lattice vibration of molecules between plate and heater.

<u>Assumptions</u>

- There is no heat loss from/to the stainless steel plate .
- The temperature of the enclosure remains constant throughout.
- The properties of the SS plate do not change with temperature
- The foil is spatially isothermal(lumped capacitance formulation)

For the above assumptions, the heating of the SS Plate can be mathematically represented as

$$E_{stored} = E_{lost} + E_{gen}$$

$$mc_p \frac{dT}{dt} = -hA(T - T_a) \qquad (1)$$

Equation (1), the left hand side represent the rate of change of enthalpy and the right hand side represents the heat transfer by convection. For substituting Nusselt number for h, Equation(1) becomes

$$\rho v C_p \frac{dT}{dt} = \frac{-Nu.K}{L}.A(T - T_a)$$

$$\rho v C_p \frac{dT}{dt} = \frac{-CRa^{0.25}KA}{L}(T - T_{a)}$$

$$\frac{dT}{dt} = \frac{-CRa^{0.25}KA}{L\rho v C_P}(T - T_a)$$

Rayleigh Number, $$Ra = \frac{g\beta \Delta T L^3}{\gamma\alpha}$$

$$\frac{dT}{T - T_a} = \frac{-CRa^{0.25}KA}{L\rho v C_p}dt$$

Initial Condition T=T_i at t=0

$$\int_{T_I}^{T} \frac{dT}{T - T_a} = \frac{-CRa^{0.25}KA}{L\rho v C_p}\int_0^t dt$$

$$\ln\left(\frac{T - T_a}{T - T_a}\right)_{T_i}^{T} = \frac{-CRa^{0.25}KAt}{LC_P \rho v}$$

$$\ln\left(\frac{T - T_a}{T - T_a}\right) - \ln\left(\frac{T_i - T_a}{T_i + T_a}\right) = \frac{-CRa^{.0.25}KAt}{LC_p \rho v} \quad (2)$$

One possibility of solving the inverse problem is to substitute various values of C in Equation(2) and determine the temperature T_i at various time instants given in the problem. With these following can be calculated

$$S(C) = \sum_{i=1}^{N}\left(T_{\exp,i} - T_{calc,i}\right)^2$$

Upon doing such an exercise for C ranging from 0.53-0.67 in the steps of 0.1. From the plot we can at the best, say that 0.53<C<0.67.The residual are plotted against the constant C and are shown in Figure 3.

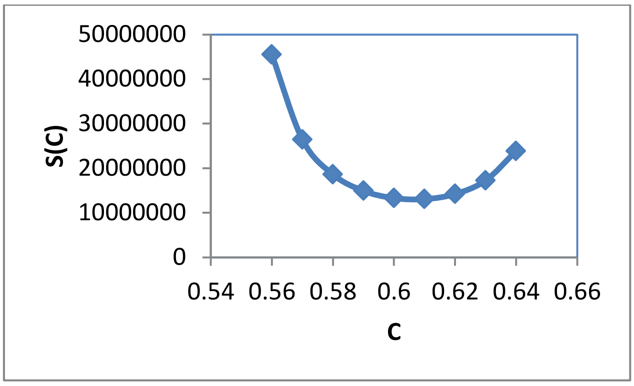

Figure 3 Residual Vs Constant C

By locally fitting a Lagragian interpolation polynomial for S(C), by employing three values of C where the residuals appear to be heading towards a minimum.These happens to be 0.57,0.6and 0.62.

$$S=\frac{(C-0.6)(C-0.62)}{(0.57-0.6)(0.57-0.62)}\times 26422412.04+\frac{(C-0..57)(C-0.62)}{(0.6-0.57)(0.6-0.62)}\times 13278078.79+\frac{(C-0..57)(C-0.6)}{(0.62-0.57)(0.62-0.6)}\times 14194282.9$$

S=9679092938**C**^{2}-11762683179**C**+3586414529

Take $\frac{ds}{dC}$ and equate it to zero to make S Stationary

$$\frac{ds}{dC}=19358185876C-11762683179=0$$

C=0.608

Therefore the best estimate of C with the level of computational intensity is 0.608.

3.2 Bayesian Approach

Bayesian inference is based on Bayes conditionality probability theorem and employs probability to characterize all forms of uncertainty in the problem. The Bayes theorem to relate the experimental data Y and the parameter is

$$P\left(\frac{x}{Y}\right)=\frac{P\left(\frac{Y}{x}\right)P(x)}{P(Y)}=\frac{P\left(\frac{Y}{x}\right)P(x)}{\int P\left(\frac{Y}{x}\right)P(x)dx} \tag{3}$$

Where P(x/Y) is called the posterior probability density function (PPDF) for which the effect is Y and x is the cause, P(Y/x) is the likelihood density function(x) is the prior density function and P(Y) is the normalizing constant. In equation (3), the first term in the RHS represent the probability of getting Y for an assumed value of x. This can be obtained from a solution to the least square residual method problem for an assumed x and convert the

$$S = \sum_{i=1}^{N} \left(Y_{\exp,i} - Y_{sim,i}\right)^2$$

into a PDF. The P(x) is prior knowledge belief about even before the measurements are made .Bayesian inference also helps us to inject a prior if it is known already from previous knowledge and this will hasten the retrieval and also helps us in obtaining sharper PPDFs signifying lower standard deviation of the estimates.

Steps involved in Bayesian approach

The Bayesian method to solve an inverse problem involves three steps

- Experimental data collection(The data is collected in the form of Temperature)
- Modeling Likelihood and priori
- Estimation of x

The first step is done by conducting experiments.in so far as the likelihood is concerned,we exploit the idea of measurement error in temperature as follow

$$Y_{measured} = Y_{simulated} + \omega$$

ω is a random variable from a normal distribution with mean"0" and standard deviation σ ,where σ is the standard deviation of the measuring instrument. Assuming that the uncertainty ω follows a normal or Gaussian distribution, the likelihood can be modeled as

$$P\left(\frac{Y}{x}\right) = \frac{1}{\left(\sqrt{2\Pi\sigma^2}\right)^n} \exp\left(\frac{\left(T - F(x)^T\right)\left(Y - F(x)\right)}{2\sigma^2}\right)$$

Where Y is a vector of dimension n i.e., n measurement are available and the F(x) is the solution to the least square residual method with the parameter vector x.

$$P\left(\frac{Y}{x}\right) = \frac{1}{\left(\sqrt{2\Pi\sigma^2}\right)^n} \exp\left(\frac{\left(-\chi^2\right)}{2}\right)$$

Where N is the total number of samples in one single experiment λ^2.

Where $$\chi^2 = \sum_{i=1}^{n} \frac{\left(Y_{meas,i} - Y_{sim,i}\right)^2}{\sigma^2}$$

Where $Y_{sim,i}$ are the simulated values of Y for an assumed x.

The posterior PDF(PPDF) then becomes

$$P\left(\frac{x}{Y}\right)=\frac{\left[\frac{1}{\left(\sqrt{2\Pi\sigma^{2}}\right)^{n}}\exp\left(\frac{-\chi^{2}}{2}\right)\right]P(x)}{\int\left[\frac{1}{\left(\sqrt{2\Pi\sigma^{2}}\right)^{n}}\exp\left(\frac{-\chi^{2}}{2}\right)\right]P(x)dx}$$

The prior probability density(x) typically follows a uniform, normal or log normal distribution. In this case of a uniform prior,P(x) is the same for all values of x,i.e,we have absolutely no selective preference. Such a prior is called a non- informative prior.

Consider P(x) follows a normal distribution with mean μ_p and standard deviation σ_p .Mathematically P(x) is given by

$$P(x)=\frac{1}{\left(\sqrt{2\Pi\sigma^{2}}\right)^{n}}\exp\left(\frac{\left(-(x-\mu_p)^{2}\right)}{2\sigma_p^{2}}\right)$$

Hence the PPDF becomes

$$P\left(\frac{x}{Y}\right)=\frac{\frac{1}{(2\Pi)^{\frac{n+1}{2}}(\sigma^{n}\sigma_p)}\exp(-)\left[\frac{\lambda^{2}}{2}+\frac{(x-\mu)^{2}}{2\sigma_p^{2}}\right]}{\left[\int\frac{1}{(2\Pi)^{\frac{n+1}{2}}(\sigma^{n}\sigma_p)}\exp(-)\left[\frac{\chi^{2}}{2}+\frac{(x-\mu)^{2}}{2\sigma_p^{2}}\right]\right]dx}$$

Therefore for every assumed value of the data vector $X(x_1,x_2,x_3,\ldots\ldots\ldots\ldots\ldots x_n)$,P(x/Y) can be worked out.

Therefore,

$$P\left(\frac{x}{Y}\right)=\frac{\exp(-)\left[\frac{\lambda^{2}}{2}+\frac{(x-\mu)^{2}}{2\sigma_p^{2}}\right]}{\left[\int\exp(-)\left[\frac{\chi^{2}}{2}+\frac{(x-\mu)^{2}}{2\sigma_p^{2}}\right]\right]dx}$$

The mean estimate of x then becomes

$$\bar{x}=\frac{\int x\exp(-)\left[\frac{\lambda^{2}}{2}+\frac{(x-\mu)^{2}}{2\sigma_p^{2}}\right]dx}{\left[\int\exp(-)\left[\frac{\chi^{2}}{2}+\frac{(x-\mu)^{2}}{2\sigma_p^{2}}\right]\right]dx}$$

Often the integral is replaced by a summation when only discrete values of x are used,

$$\bar{x} = \frac{\sum_i x_i \exp(-)\left[\frac{\lambda^2}{2} + \frac{(x-\mu)^2}{2\sigma_p^2}\right]\Delta x_i}{\sum \left[\exp(-)\left[\frac{\lambda^2}{2} + \frac{(x-\mu)^2}{2\sigma_p^2}\right]\right]\Delta x_i}$$

$$\bar{x} = \frac{\sum_i x_i \exp(-)\left[\frac{\lambda^2}{2} + \frac{(x-\mu)^2}{2\sigma_p^2}\right]}{\sum \left[\exp(-)\left[\frac{\lambda^2}{2} + \frac{(x-\mu)^2}{2\sigma_p^2}\right]\right]}$$

$$\sigma_x^2 = \frac{\sum_i (x_i - \bar{x})\exp(-)\left[\frac{\lambda^2}{2} + \frac{(x-\mu)^2}{2\sigma_p^2}\right]}{\sum \left[\exp(-)\left[\frac{\lambda^2}{2} + \frac{(x-\mu)^2}{2\sigma_p^2}\right]\right]} \quad (4)$$

Now we can use this framework to estimate the constant C in the Nusslet number.in Equation(4) is the standard deviation of the estimated parameter, which is very diagnostic of the potency of the estimation process.Table 2 shows the Mean and Standard Deviation for single parameter using Bayesian method(With and Without Priori).Figure 4 and Figure 5 shows the typical PPDF for the constant C in the Nusslet number correlation.Table 5 shows the comparision of optimization result with standard correlation.

Table 2 Mean and Standard Deviation for single parameter using Bayesian method

	Bayesian Approach	
	Without Prior	With Prior
Mean (C)	0.608167	0.60770467
Standard Deviation	0.012452	0.00479451

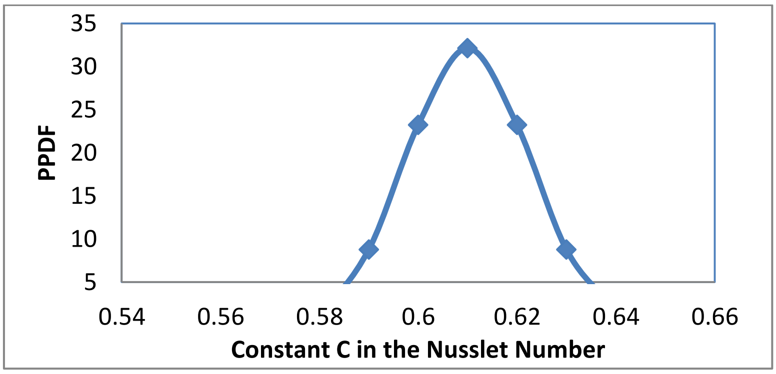

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Figure 4 PPDF of Constant C in the Nusslet Number using Bayesian Method

(Without Prior)

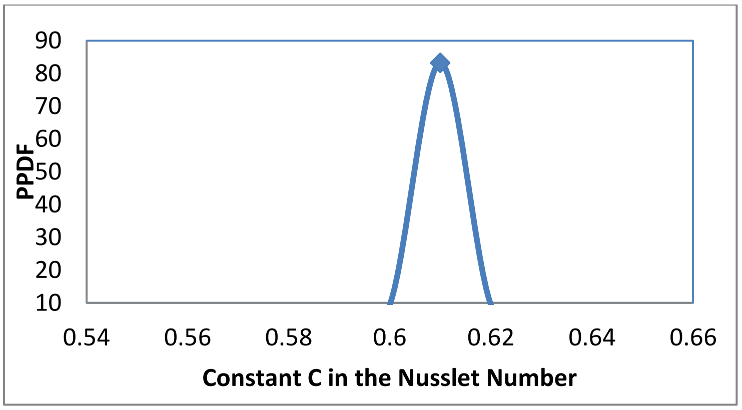

Figure 5 PPDF of Constant C in the Nusslet Number using Bayesian Method

(With Prior)

Table 3 Comparison of Optimization Results with Yousef Correlation

SI No.	Correlation	C
1	Yousef Correlation	0.62
2	Least square residual Optimization Method	0.61
3	Bayesian Approach	0.61

4. Conclusion

Steady heat transfer laminar natural convection experiment in horizontal heated plate facing upward have been done for a highly polished stainless steel plate to retrieve single parameter (constant value C in the nusslet number) by using least square residual method and Bayesian appproach and compared with the value available from yousef correlation and found to be in good agreement.

Nomenclature

A- Surface Area

C- Constant in the Nusslet Number

C_p- Specific Heat at Constant Pressure

K- Thermal Conductivity of the Fluid

L- Characteristic Dimension of the Surface

m- Mass of the Material

Nu- Nusslet Number

ρ - Density of Fluid

S(c)- Residual

T - Plate Temperature

T_a- Ambient Temperature

Ti- Instantaneous Temperature

Greek Symbols

σ - Stefan-Botzmann Constant

γ - Kinematic Viscosity of the Fluid

β -Coefficient of Volumetric Expansion of the Fluid

α - Thermal Diffusivity of the Fluid

σ_p - Standard Deviation

μ_p - Mean

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