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ESTIMATION OF HEAT FLUX FOR UNSTEADY STATE HEAT TRANSFER OVER GAS TURBINE WALL, STEEL REHEAT FURNACE AND CERAMIC KILN

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Abstract

Least Square approach is employed in many branches of engineering and science. In case of heat transfer problems, least square approach can be used to estimate heat flux, or can be used to determine important thermal properties such as the thermal conductivity or heat capacity of solids. This, in turn, could be useful for cases where direct measurement of heat flux is not possible due to extremely high temperatures. This type of situation is encountered at surface of a space re-entry vehicle or in a gas turbine engine combustion chamber or high-pressure turbine or furnace. The significance of heat flux and temperature estimation provides vital insight on maximum operating temperature of the solid. The work presented in this paper aims to find out the heat flux and heat transfer coefficient over a flat plate for unsteady phenomenon of heat transfer by analyzing a simple one-dimensional model of a high-pressure gas engine wall, with constant thermal properties and no heat generation within the metal. One side of the wall is exposed to gas path hot air and the other side is exposed to cooling air bled off from the fan in air duct. A sensor at the cool surface of the metal measures temperatures over a range of time. The goal is to use the measured temperatures and the Least Square analysis to estimate the amount of heat flux required on the cool side of the wall to keep the metal on the gas path side below the maximum operating temperature in order to avoid incomplete combustion.

Keywords: Unsteady state heat transfer; Least Square Approach; Gas Turbine; Steel Reheat Furnace; Ceramic Kiln.

1. INTRODUCTION

1.1 Convective heat transfer

Convective heat transfer is the mode of heat transfer generally observed in fluids. While in conductive heat transfer, energy is transferred by vibrations at molecular level and radiative heat transfer involves energy transfer by electromagnetic radiation, convective heat transfer occurs by bulk motion of fluids. Convective heat transfer is observed in numerous examples of naturally occurring fluid flow, such as wind, oceanic

Vol.1 Issue.7, November 2013. Pgs: 204-216

currents, and movements within the Earth's mantle. Convection is also used in engineering practices to provide desired temperature changes, as in heating of homes, industrial processes, cooling of equipment, etc.

1.2 Transient conduction (Unsteady-state Conduction)

In general, during any period in which temperatures are changing *in time* at any place within an object, the mode of thermal energy flow is termed *transient conduction.* Another term is "non-steady-state" conduction, referring to time-dependence of temperature fields in an object. Non-steady-state situations appear after an imposed change in temperature at a boundary of an object. They may also occur with temperature changes inside an object, as a result of a new source or sink of heat suddenly introduced within an object, causing temperatures near the source or sink to change in time.

When a new perturbation of temperature of this type happens, temperatures within the system will change in time toward a new equilibrium with the new conditions, provided that these do not change. After equilibrium, heat flow into the system will once again equal the heat flow out, and temperatures at each point inside the system no longer change. Once this happens, transient conduction is ended, although steady-state conduction may continue if there continues to be heat flow.

If changes in external temperatures or internal heat generation changes are too rapid for equilibrium of temperatures in space to take place, then the system never reaches a state of unchanging temperature distribution in time, and the system remains in a transient state.

An example of a new source of heat "turning on" within an object which causes transient conduction is an engine starting in an automobile. In this case the transient thermal conduction phase for the entire machine would be over, and the steady state phase would appear, as soon as the engine had reached steady-state operating temperature. In this state of steady-state equilibrium, temperatures would vary greatly from the engine cylinders to other parts of the automobile, but at no point in space within the automobile would temperature be increasing or decreasing. After establishment of this state, the transient conduction phase of heat transfer would be over.

An example of transient conduction which does not end with steady-state conduction, but rather no conduction, occurs when a hot copper ball is dropped into oil at a low temperature. Here the temperature field within the object begins to change as a function of time, as the heat is removed from the metal, and the interest lies in analyzing this spatial change of temperature within the object over time, until all gradients disappear entirely (the ball has reached the same temperature as the oil). Mathematically, this condition is also approached exponentially; in theory it takes infinite time, but in practice it is over, for all intents and purposes, in a much shorter period. At the end of this process with no heat sink but the internal parts of the ball (which are finite), there is no steady state heat conduction to be reached. Such a state never occurs in this situation, but rather the end of the process is when there is no heat conduction at all.

Analysis of non-steady-state conduction systems is more complex than steady-state systems, and (except for simple shapes) calls for the application of approximation theories, and/or numerical analysis by computer. One popular graphical method involves the use of Heisler Charts.

Occasionally transient conduction problems may be considerably simplified if regions of the object being heated or cooled can be identified, in which thermal conductivity is very much greater than that for heat paths leading into the region. In this case, the region with high conductivity can often be treated in the lumped capacitance model, as a "lump" of material with a simple thermal capacitance consisting of its aggregate heat capacity. Such regions show no temperature variation across their extent during warming or cooling (as compared to the rest of the system) due to their far higher conductance. During transient conduction, therefore, their temperature changes uniformly in space, and as a simple exponential in time.

An example of such systems is those which follow "Newton's law of cooling" during transient cooling (or the reverse during heating). The equivalent thermal circuit consists of a simple capacitor in series with a resistor. In such cases, the remainder of the system with high thermal resistance (comparatively low conductivity) plays the role of the resistor in the circuit.

1.3 Applications of the Unsteady State analysis

a) The design of boilers, condensers, evaporators, heaters, refrigerators and heat exchangers, requires considerations of the amount of heat to be transmitted as well as the rate at which heat is to be transferred.
b) The successful operation of equipment component such as turbine blades, the walls of combustion chambers etc. depends on the cooling rate, in order to avoid their metallurgical failure.
c) A heat transfer analysis must also be accounted in the design of electronic components, electric machines, transformers, and bearing to avoid the overheating and damage of equipment.
d) Forced convective heat transfer of nanofluids in minichannels.
e) Laminar Forced Convection Heat Transfer in the Combined Entry Region of Non-Circular Ducts.
f) Simultaneous unsteady state diffusion and heat conduction in the absence of the Soret Effect. Under these conditions the problem reduces to diffusion with a variable diffusion coefficient which is produced by the non-uniformity of the temperature field.
g) It is shown that the variability of temperature is unlikely to be important for most solids and liquids, however in the case of gases it may have a significant effect on both the flux and the concentration profile.

2. Problem Formulation

The direct formulation of the heat conduction problem is given as such:

$$
\begin{array}{llll}
\dfrac{\partial^2 T}{\partial x^2} = \dfrac{1}{\alpha}\dfrac{\partial T}{\partial t} & in & 0 < x < L & t > 0 \\
1)\quad -k\dfrac{\partial T}{\partial x} = q(t) & @ & x = 0 & t > 0 \\
T = T_{hot} = T_{\max} & @ & x = L & t > 0 \\
T = T_{\max} & for & t = 0 & 0 \le x \le L
\end{array}
$$

In the direct formulation the surface heat flux at x = 0 is considered known. In the Least Square approach, the surface heat flux is considered unknown. For Least Square approach formulation of the problem can be given as:

$$
\begin{array}{llll}
\dfrac{\partial^2 T}{\partial x^2} = \dfrac{1}{\alpha}\dfrac{\partial T}{\partial t} & in & 0 < x < L & t > 0 \\
2)\quad -k\dfrac{\partial T}{\partial x} = q(t) = ? & @ & x = 0 & \\
T = T_{hot} = T_{\max} & @ & x = L & \\
T = T_{\max} & for & t = 0 & 0 \le x \le L
\end{array}
$$

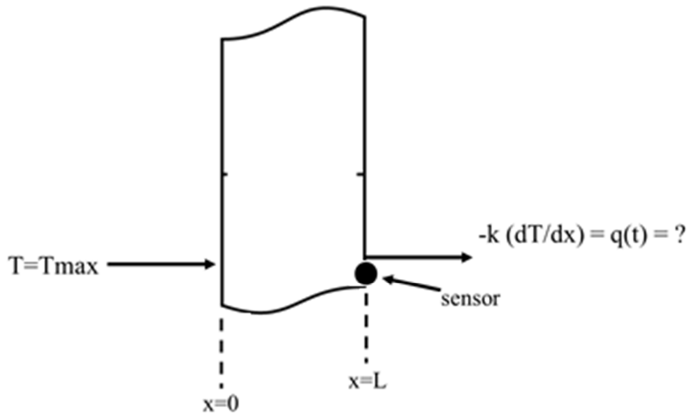

Figure 1: Temp variation across Wall

Vol.1 Issue.7,
November 2013.
Pgs: 204-216

3. Methodology

The main difficulty encountered when attempting to solve Least Square approach is that the solutions are very sensitive to changes in the input data and thus may not be unique. This can be a result of measurement and modeling errors. This places the Least Square approach in the class of mathematical problems called the ill-posed problems. This is because the solution of the Least Square approach does not satisfy the general requirement of existence, uniqueness, and stability with small changes to the input data. In contrast, direct heat conduction problems are well posed because they satisfy the requirements of existence, uniqueness, and stability of the solution.

Various methods for solving Least Square approach have been proposed and executed but many were unstable or not useful for practical applications. To generate a successful solution to a Least Square approach one generally needs to transform the problem into a well-posed approximate solution. One good way to do this is to transform the problem into a least squares problem. This transformation requires that the solution minimize the least squares norm, rather than make it zero, which guarantees the existence of an inverse solution.

To solve the problem by this method we require that the estimated temperatures match the measured temperatures as closely as possible over a specified time domain. The estimated temperatures are computed from the solution of the direct problem by using the estimated heat flux components, whereas the measured temperatures are recorded using a sensor placed at a "strategic" location that will provide the least measurement error. To ensure optimal matching between the measured and estimated temperatures we require that the least squares norm is minimized with respect to each of the unknown heat flux components. Here is the least squares norm modified by the addition of a zeroth-order regularization term:

1) $$S(\hat{\mathbf{q}}) = \sum_{j=1}^{M} \left[Y_j - \hat{T}_j(\hat{\mathbf{q}})\right]^2 + \alpha^* \sum_{j=1}^{M} \hat{q}_j^2$$

where $\hat{\mathbf{q}} \equiv [\hat{q}_i \quad for \quad i = 1,2,...,M]$ and the superscript ^ denotes the estimated values. The other quantities are defined by

$S(\hat{\mathbf{q}})$ = sum of squares

$\hat{q}_j \equiv \hat{q}(t_j)$ = estimated surface heat flux at the boundary

$Y_j \equiv Y(t_j)$ = measured temperature at surface x = 0, at times t_j

$\hat{T}_j(\hat{\mathbf{q}})$ = estimated temperature at the surface x = 0 at times t = t_j computed by using estimated heat flux, $[\hat{q}_i, i = 1,2,...,M] = \hat{\mathbf{q}}$

α^* = the regularization parameter > 0

In equation 3, the first term is the traditional least squares. The second term is the zero-order regularization term used to reduce instability or oscillations that are inherent in the solution of ill-posed problems. If the regularization parameter goes to zero the solution exhibits oscillatory behavior and becomes unstable if a large number of parameters are to be estimated. If the regularization parameter is a large value, the solution is damped and deviates from the exact solution. Studies have shown that a relatively wide range of alpha star can be used, and depending on the value of the standard deviation of measurement errors, whereas zero measurement error is assumed throughout the analysis. The optimum value of the regularization parameter ranged from 10E-2 to 10E-4. During the current numerical analysis median of the optimum value range, 10E-3.

What we would like to do next is minimize the least squares equation by differentiating it with respect to each of the unknown heat flux components and setting the resulting expression equal to zero. Doing so yields:

2) $$\frac{\partial S(\hat{\mathbf{q}})}{\partial \hat{q}_i} = 2\sum_{j=1}^{M} \frac{\partial \hat{T}_j(\hat{\mathbf{q}})}{\partial \hat{q}_i}\left[\hat{T}_j(\hat{\mathbf{q}}) - Y_j\right] + 2\alpha^* \sum_{j=1}^{M} \hat{q}_j \frac{\partial \hat{q}_j}{\partial \hat{q}_i} = 0$$

where i = 1,2,…,M and

3) $$\frac{\partial \hat{q}_j}{\partial \hat{q}_i} = \begin{cases} 0 & for \quad i \neq j \\ 1 & for \quad i = j \end{cases}$$

Equation 4, can be rearranged as

4) $$\sum_{j=1}^{M} \frac{\partial \hat{T}_j(\hat{\mathbf{q}})}{\partial \hat{q}_i}\left[Y_j - \hat{T}_j(\hat{\mathbf{q}})\right] = \alpha^* \sum_{j=1}^{M} \hat{q}_j \frac{\partial \hat{q}_j}{\partial \hat{q}_i}$$

where i = 1,2,…,M and

5) $$\frac{\partial \hat{T}_j(\hat{\mathbf{q}})}{\partial \hat{q}_i} = \frac{\partial \hat{T}_j(\hat{q}_1, \hat{q}_2, ..., \hat{q}_M)}{\partial \hat{q}_i} \equiv X_{ji} = \text{sensitivity coefficients w.r.t. } \hat{q}_i$$

Equation 6 can be written in the matrix form as

6) $$\mathbf{X}^T(\mathbf{Y} - \mathbf{T}) = \alpha^* \mathbf{q}$$

where the vectors are given by

7) $$\mathbf{T} = \begin{bmatrix} \hat{T}_1 \\ \hat{T}_2 \\ \vdots \\ \hat{T}_M \end{bmatrix}, \quad \mathbf{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_M \end{bmatrix}, \quad \mathbf{q} = \begin{bmatrix} \hat{q}_1 \\ \hat{q}_2 \\ \vdots \\ \hat{q}_M \end{bmatrix}$$

and the sensitivity matrix **X** with respect to **q** is written explicitly as

8) $$\mathbf{X} \equiv \frac{\partial \mathbf{T}}{\partial \mathbf{q}^T} = \begin{bmatrix} \frac{\partial T_1}{\partial q_1} & \frac{\partial T_1}{\partial q_2} & \cdots & \frac{\partial T_1}{\partial q_M} \\ \frac{\partial T_2}{\partial q_1} & \frac{\partial T_2}{\partial q_2} & & \frac{\partial T_2}{\partial q_M} \\ \vdots & & & \vdots \\ \frac{\partial T_M}{\partial q_1} & \frac{\partial T_M}{\partial q_2} & \cdots & \frac{\partial T_M}{\partial q_M} \end{bmatrix}$$

In this sensitivity matrix the terms above the diagonal must be zero because the temperatures $\hat{T}_i$ calculated at any instant of time t_i must be independent of the future heat fluxes, $\hat{q}_j, j > i$.

In order to solve equation 8 it is desirable to express it in a more convenient form. This is achieved by expanding the estimated temperatures in a Taylor series with respect to an arbitrary value of the heat flux like so:

9) $$\hat{T}_j = \hat{T}_{0j} + \sum_{k=1}^{M} \frac{\partial \hat{T}_j}{\partial \hat{q}_k}(\hat{q}_k - \hat{q}_0)$$

If we choose the arbitrary point to be 0, then the equation reduces to

10) $$\hat{T}_j = \sum_{k=1}^{M} \frac{\partial \hat{T}_j}{\partial \hat{q}_k} \hat{q}_k$$

or in matrix form as

11) $$\mathbf{T} = \frac{\partial \mathbf{T}}{\partial \mathbf{q}^T} \mathbf{q} \equiv \mathbf{Xq}$$

If we substitute equation 12 into equation 6 we get

12) $$\sum_{j=1}^{M} \frac{\partial \hat{T}_j(\hat{\mathbf{q}})}{\partial \hat{q}_i} \left[Y_j - \sum_{k=1}^{M} \frac{\partial \hat{T}_j}{\partial \hat{q}_k} \hat{q}_k \right] = \alpha^* \sum_{j=1}^{M} \hat{q}_j \frac{\partial \hat{q}_j}{\partial \hat{q}_i}$$

The matrix form of this equation is

13) $$\mathbf{X}^T (\mathbf{Y} - \mathbf{Xq}) = \alpha^* \mathbf{q}$$

which can be rearranged as

14) $$\mathbf{q} = \left(\mathbf{X}^{\mathrm{T}} \mathbf{X} + \alpha^* \mathbf{I} \right)^{-1} \mathbf{X}^{\mathrm{T}} \mathbf{Y}$$

Equation 16 is the formal solution of the Least Square approach for the unknown heat flux over the period $0 < t < t_f$. Once the sensitivity coefficients, the regularization parameter, and the measured temperatures are available, we can directly compute the heat flux. Since regularization parameter is already selected to be 10E-3 and the measured temperatures are known, all that is left is the calculation of the sensitivity coefficients.

Because the direct problem associated with the Least Square approach is linear, we can use Duhamel's theorem to solve the direct problem containing a time dependent boundary condition. Duhamel's theorem states:

15) $$T(x,t) = \int_{\lambda=0}^{t} q(\lambda) \frac{\partial \phi(x, t-\lambda)}{\partial t} d\lambda$$

where $\phi(x,t)$ is the solution to the following auxiliary problem (obtained from the direct problem formulation):

16) $$\begin{aligned} \frac{\partial^2 \phi}{\partial x^2} &= \frac{1}{\alpha} \frac{\partial \phi}{\partial t} && in \quad 0 < x < L \\ -k \frac{\partial \phi}{\partial x} &= 1 && @ \quad x = 0 \\ \phi &= 1 && @ \quad x = L \\ \phi &= 1 && for \quad t = 0 \end{aligned}$$

This auxiliary problem is a 1 dimensional linear transient problem with no heat generation and nonhomogeneous boundary conditions. The auxiliary problem can be split into a set of simpler problems containing a nonhomogeneous steady state problem and a homogeneous transient problem, which can be solved by the method of separation of variables. The steady state problem is given by:

17) $$\begin{aligned} \frac{d^2 \phi}{dx^2} &= 0 && in \quad 0 < x < L \\ -k \frac{d\phi}{dx} &= 1 && @ \quad x = 0 \\ \phi &= 1 && @ \quad x = L \end{aligned}$$

and the homogeneous problem is given by:

18)
$$\frac{\partial^2\phi}{\partial x^2}=\frac{1}{\alpha}\frac{\partial\phi}{\partial t} \quad in \quad 0<x<L$$
$$k\frac{\partial\phi}{\partial x}=0 \quad @ \quad x=0$$
$$\phi=0 \quad @ \quad x=L$$
$$\phi_h=\phi(x)-\phi_s=f^*(x) \quad for \quad t=0,0<x<L$$

Then, the solution of the original auxiliary problem is determined from

19) $$\phi(x,t)=\phi_s(x)+\phi_h(x,t)$$

The steady state problem can be integrated twice, with the use of the boundary conditions, to yield:

20) $$\phi_s(x)=1+\frac{1}{k}(L-x)$$

The homogeneous problem is the familiar transient 1-D slab problem, whose solution can be written immediately as:

21) $$\phi_h(x,t)=\sum_{m=0}^{\infty}e^{-\alpha\beta_m^2 t}\frac{1}{N(\beta_m)}X(\beta_m,x)\int_{x'=0}^{L}X(\beta_m,x)f^*(x')dx'$$

Now we can determine the Eigen functions, norms of eigenvalues, and the eigenvalues, which are respectively:

22) $$X(\beta_m,x)=\cos\beta_m x,\quad \frac{1}{N(\beta_m)}=\frac{2}{L},\quad \cos\beta_m L=0$$

where the eigenvalues, beta, are the positive roots of the third equation. Substituting into the homogeneous solution we get:

23) $$\phi_h(x,t)=\frac{2}{L}\sum_{m=0}^{\infty}e^{-\alpha\beta_m^2 t}\cos\beta_m x\int_{x'=0}^{L}1-\left[1-\frac{1}{k}(L-x')\right]\cos\beta_m x'dx'$$

If we perform the integrations (which are fairly lengthy and will not be shown here) we get:

24) $$\phi_h(x,t)=-\frac{2}{kL}\sum_{m=0}^{\infty}e^{-\alpha\beta_m^2 t}\frac{\cos\beta_m x}{\beta_m^2}$$

25) $$\beta_m=\frac{(2m+1)}{L}\frac{\pi}{2},\quad m=0,1,2,...$$

Thus, the overall solution to the nonhomogeneous auxiliary problem is

26) $$\phi(x,t)=1+\frac{1}{k}(L-x)-\frac{2}{kL}\sum_{m=0}^{\infty}e^{-\alpha\beta_m^2 t}\frac{\cos\beta_m x}{\beta_m^2}$$

Now that we have solved the auxiliary problem we can return to Duhamel's theorem and use the auxiliary solution. Duhamel's theorem can be written in the alternative form as:

27) $$T(x,t)=\int_{\lambda=0}^{t}q(\lambda)\frac{\partial\phi(x,t-\lambda)}{\partial\lambda}d\lambda$$

since

28) $$\frac{\partial\phi(x,t-\lambda)}{\partial t}=-\frac{\partial\phi(x,t-\lambda)}{\Delta\lambda}$$

The integral in equation 29 can be discretized as:

Vol.1 Issue.7, November 2013. Pgs: 204-216

29)
$$T(x,t_M) = \sum_{n=1}^{M} \frac{\phi(x,t_M-\lambda_{n-1})-\phi(x,t_M-\lambda_n)}{\Delta\lambda}\Delta\lambda$$
$$= \sum_{n=1}^{M} q_n\left[\phi(x,t_{M-(n-1)})-\phi(x,t_{M-n})\right]$$

which is written more compactly as:

30)
$$T_M = \sum_{n=1}^{M} q_n \Delta\phi_{M-n} \quad n = 1,2,\ldots,M$$

In matrix form this is written as:

31)
$$\begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ \vdots \\ T_M \end{bmatrix} = \begin{bmatrix} \Delta\phi_0 & & & & \\ \Delta\phi_1 & \Delta\phi_0 & & \mathbf{0} & \\ \Delta\phi_2 & \Delta\phi_1 & \Delta\phi_0 & & \\ \vdots & \cdots & \cdots & \ddots & \\ \Delta\phi_{M-1} & \cdots & \cdots & \Delta\phi_1 & \Delta\phi_0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ \vdots \\ q_M \end{bmatrix}$$

If we recall the definition of **X** from equation 10, we can see that the coefficient matrix must be the sensitivity matrix **X**. Thus,

32)
$$\mathbf{X} \equiv \mathbf{X}_{ij} = \begin{bmatrix} \Delta\phi_0 & & & & \\ \Delta\phi_1 & \Delta\phi_0 & & \mathbf{0} & \\ \Delta\phi_2 & \Delta\phi_1 & \Delta\phi_0 & & \\ \vdots & \cdots & \cdots & \ddots & \\ \Delta\phi_{M-1} & \cdots & \cdots & \Delta\phi_1 & \Delta\phi_0 \end{bmatrix}$$

Where $\Delta\phi_i \equiv \phi_{i+1} - \phi_i$ *and* $\phi_i \equiv \phi(x,t_i)$. ϕ represents the temperature rise in the solid for a unit step increase in the surface heat flux, and can be computed directly from the solution to the auxiliary problem, which is evaluated at the sensor location, x = 0, over the range of times. Now that we have the measured temperatures, the sensitivity coefficients, and the regularization parameter, we can determine the unknown surface heat flux vector from equation 16.

4. Results And Discussions

4.1 Wall of High Pressure Gas Turbine with Assumed Temperature History

The solution to the Least Square approach of the problem was executed in MATLAB. Since temperatures were not available over a range of times, a linear temperature distribution of $T_{cold} = at + b$ was assumed. Therefore, if we assume that the temperature on the cool side of the wall is equal to the hot metal temperature of 2000^{o}C at the initial time t = 0, and the temperature on the cool side is equal to the steady state temperature 1200^{o}C at the final temperature, we get an equation for the temperature on the cool side of the wall as a function of time:

1)
$$T_{cool} = \left(\frac{T_{s.s.} - T_{hot}}{t_{final}}\right)t + T_{hot}$$

Assuming that steady state temperature was reached after 60 seconds, and that temperature measurements were taken at four evenly spaced times of 15 seconds. Using these conditions for the starting and ending times, temperatures, and sensor measurements, the MATLAB program produced a surface heat flux vector of

2) $$q = \begin{bmatrix} 329.0700 \\ 255.3810 \\ 182.1354 \\ 104.9806 \end{bmatrix}$$

where q is measured in W/m^2 and the four points correspond to t = 15, 30, 45, 60 seconds at cool side temperatures of 1800, 1600, 1400, and 1200^{o}C respectively (see fig 5.1). The results are somewhat promising. They showed a heat flux vector decreasing with time. At first this seemed counter-intuitive. To validate if this problem is coded correctly in MATLAB it has been solved example 5-3 in the textbook. This problem had a time varying heat flux of t, which, when plotted, gives a line with a slope of one. The resulting graph from the least square approach solution showed a time varying heat flux that was parallel to the original heat flux but offset slightly under it (see fig. 5.2). This proved that the MATLAB program was coded correctly. However, I knew that I was still getting incorrect results for the project, although they were of the correct trend. We would expect to see the heat flux vector decrease with time because more heat flux is needed at the beginning of the problem than at the end. If we plot Temperature vs. x, we can see that for early times the temperature stays fairly constant along the slab until we approach the cool side, and then temperature drops off sharply. As time progresses, the temperature in the slab approaches the straight line representing the linear temperature distribution at steady state. Since the heat flux is the negative of the derivative of the temperature with respect to x, the slope of the temperature profile represents the heat flux. As described here, the slope of the temperature profile at x = 0 (cool side) becomes less and less negative as we approach the steady state time value. If we solve the steady state problem given as:

3) $$\begin{aligned} \frac{d^2T}{dx^2} &= 0 \quad in \quad 0 < x < L \\ T &= 1200 \quad @ \quad x = 0 \\ T &= 2000 \quad @ \quad x = L \end{aligned}$$

we get a solution of:

4) $$T(x) = \frac{800}{L} x + 1200$$

If we take the first derivative of T with respect to x and multiply by -k to obtain the heat flux we get:

5) $$q = -k \frac{800}{L}$$

which, when substituting in, gives a value of 1235200 Watt/m^2. It is thus observed that there is a deviation from the numerical simulation result. Further, when the input to the problem was changed (number of points, total time elapsed, etc), the final value for heat flux remained the same. This indicated that the assumption of linear temperature drop in the problem formulation created the error in the final results. This is probably due to the fact that creating the measured temperature vector (for lack of any better information) and forcing it to a fixed value at the final time. Also, the measured temperature vector is not necessarily linear, and might be more of a parabolic or hyperbolic shape. This assumption of a linear temperature drop could also produce incorrect results.

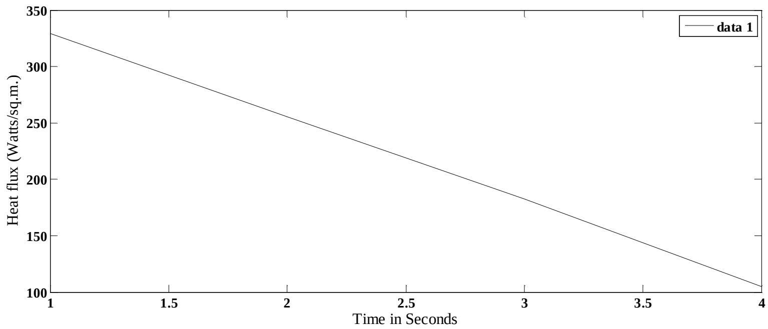

Figure 2: Plot of heat flux distribution with time for gas turbine

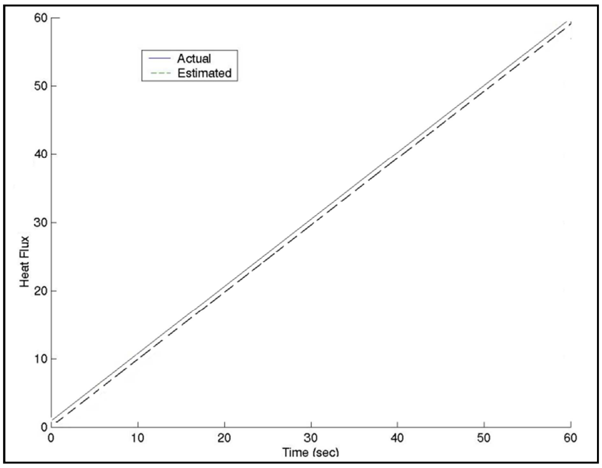

Figure 3: Comparison of estimated and actual heat flux

On another note, the regularization parameter was varied within the suggested optimum values to determine its effect on the stability of the solution. It was observed that varying this parameter had negligible effect on the results. The major factor was the value used for the final time. If the value was large, as it was for 60 seconds, the summation terms approached zero very rapidly due to the combined effect of the time and the squared eigenvalue. Since this is the case, the sensitivity coefficients all approached zero, except for the ones corresponding to t = 0, which appear on the diagonal. Since it is desirable to have large, uncorrelated values for the sensitivity coefficients, this poses a problem. As time was decreased, to determine its effect on the solution, a significant change was not seen until the final time was one second or lower. For these small times, the sensitivity coefficient matrix was zero only above the diagonal, which is correct, but some of the values were small, which is undesirable.

4.2 Steel Reheat Furnace- A Validation with Literature

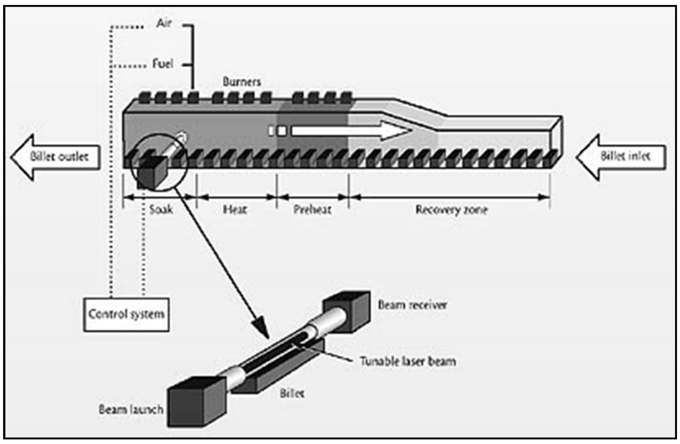

Figure 4: Schematic of a steel reheat furnace

Vol.1 Issue.7,
November 2013.
Pgs: 204-216

A steel reheat furnace heats the steel charge to minimum temperature consistent with achieving the correct temperature and metallurgical properties at the finishing stands of the mill. Uniformity of the temperature within the load, minimization of local temperature gradients, avoidance of surface defects such as skid marks, overheating marks and oxidation scale represent the characteristics of the ideal product of the reheating operation.

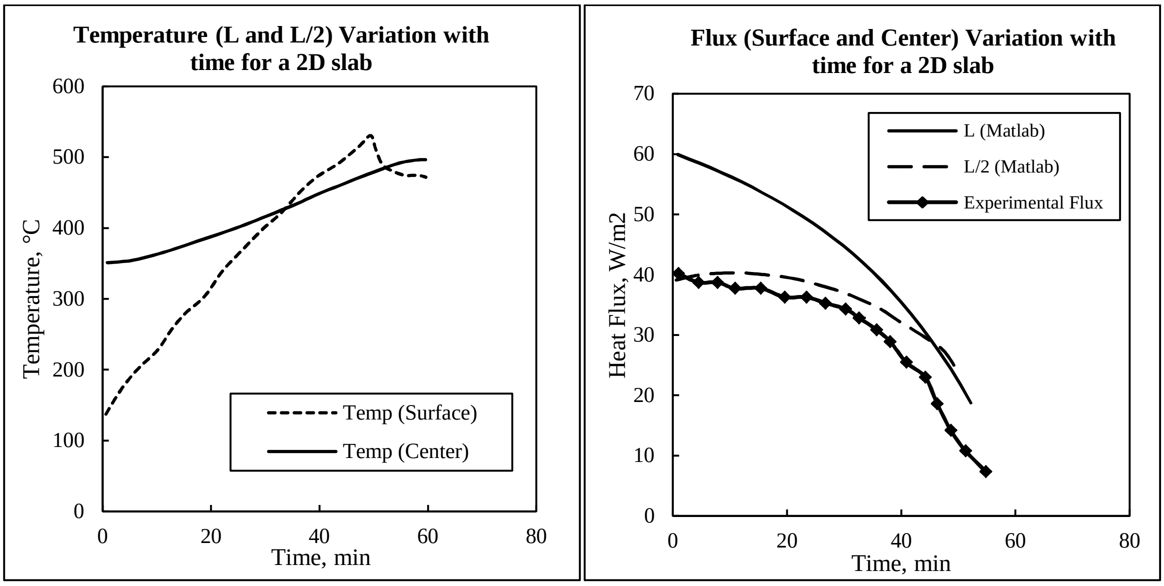

Figure 5 and 6: Temperature and Flux variation for a 2D slab

The heating operation for an individual slab usually takes between two and three hours during which lime the slab is moved inside the furnace in a prescribed manner dependent upon the type of furnace and operation for example beam or pusher furnace.

Within the furnace the slab experiences different heat transfer conditions dependent upon the surrounding atmosphere. The external conditions directly modify the heat flux to the stock via the gas temperature and the heat transfer coefficients; hence they are fundamental in determining steel properties and phenomena such as scale formation.

The fully coupled procedure has been applied, as a first demonstration, to the simple application of a two dimensional furnace a slab. Slabs of dimensions 1.2m by 0.4m enter a 12.5m long furnace zone. The slabs are assumed to be at an initial temperature of 350K. The reheating process for each slab lasts one hour and stops six times along the furnace for durations of 12 minutes each time. The furnace zone is heated by hot combustion products being exhausted from a single burner. The hot gases pass through the furnace and exit from an outlet on the floor of the furnace. The walls of the furnace are considered and a value of 0.8 is prescribed.

4.3 Simulation of Ceramic Furnaces (Kiln) – A validation with Literature

Manufacture of ceramics involves firstly control of grain size, moisture and mixing of natural inorganic substances. Then it is formed and heat treated into a sintered state. Although, industrial ceramics are produced using 3 highly sophisticated furnaces with on-line controls, small potters in developing nations like India still use traditional pottery kilns to produce ceramic ware. Mostly these kilns are wood/ coal fired. In urban/ semi-urban areas in India many kilns are fired with liquefied petroleum gas (LPG) and produce up to 1000 kg of ware in one firing. 3 Typically the small kilns are of about 1 to 4 m in volume. The firing cycles vary from 6 to 50 hour duration.

The firing cycle, which is the temperature vs. time cycle for manufacturing the ceramic material, has typically following stages:

a) Very slow firing (or Smoking): Ambient to 200°C @1°C/ min, during which residual moisture leaves.
b) Slow firing: 200 to 500°C @ 2°C/ min, here organic substances are carbonized and combusted (material strength reduces).
c) Medium firing: 500 to 700°C @ 3°C/ min, here crystalline water of clay minerals leaves, and much energy is needed.
d) Fast firing: 700°C to the maximum temperature @ 4 to 5°C/min: here the organic substances carbonized earlier are subjected to oxidation beyond 800 C and soot removal takes place. Sufficient air is needed for this. If left to cool after reaching 800°C, 'biscuit ware' (unglazed earthenware or terracotta) are produced. If the temperature is taken beyond 800 C then active sintering takes place, hard & dense ware are produced. Temperature must be raised uniformly. If the body is large, there are temperature gradients, and the wares need to be maintained at maximum temperature for some time (soaking).
e) Cooling: Controlled cooling is done at a predetermined rate.

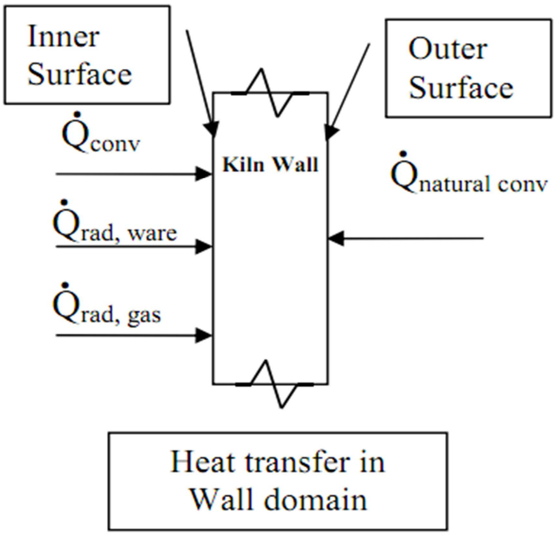

Figure 7: Heat transfer in wall Domain of a ceramic Furnace

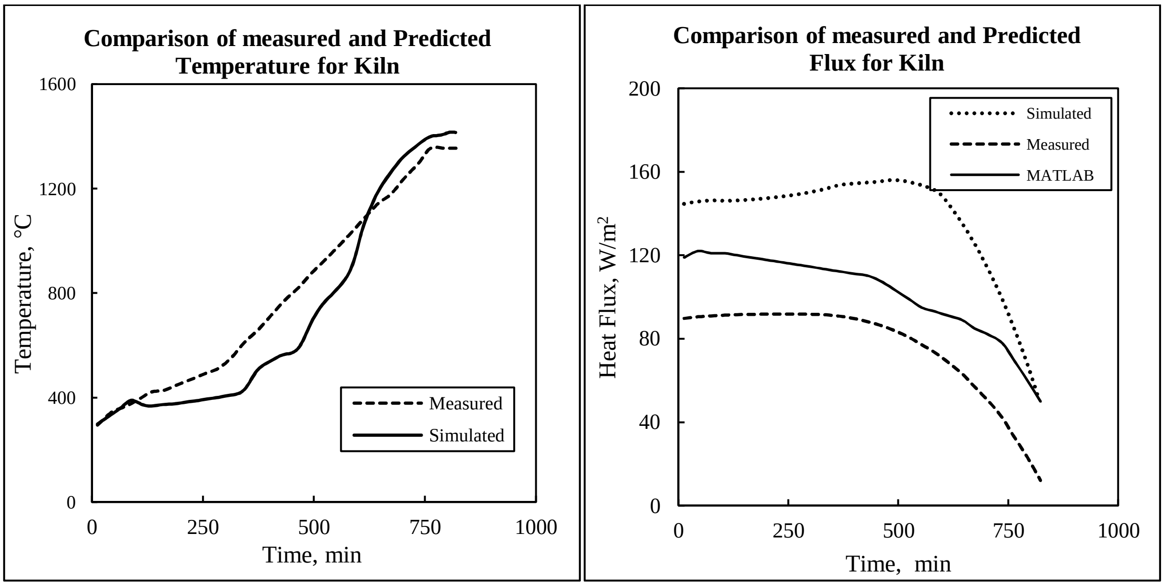

Figure 8 and 9: Temperature and Flux variation for a ceramic Kiln

Vol.1 Issue.7,
November 2013.
Pgs: 204-216

5. Conclusion

Least square approach of the problems can used to obtain a good approximation of the unknown heat flux. Heat flux could be estimated more accurately using softwares such as ANSYS or PATRAN, or using a different representation, such as a parabolic or hyperbolic shape, for the measured temperature vector.

Accurate heat flux estimation can be obtained if the temperature profile is known accurately. With continuous monitoring of temperature distribution and the properties of the material, the heat flux corresponding to temperature changes could be obtained, and thereby heat transfer coefficient at every instant can be estimated. By real time monitoring of heat transfer coefficient, the designing of equipment subject to such flow conditions could be optimized.

References

1. Atefi. G., Ganjehkaviri. A and Addous. M.A., "*Analytical Solution of Temperature Field in Hollow Cylinder under Time Dependent Boundary Condition Using Fourier series*", American Journal of Engineering and Applied Sciences 1(2): 141-148, 2008 ISSN 1941-7020
2. Venturino .M.,and Rubini. P.A., "*Coupled Fluid Flow and Heat Transfer analysis of Steel Reheat F*urnaces", 3rd European Conference on Industrial Boilers and Furnaces, Vol 1, INFUB, 1995, ISBN 972-8034-02-4
3. Gokhale. S, Ravi.M.R, Dhar.P.L, and Kaushik.S.C., "*Simulation of Ceramic Furnaces using One Dimensional model of Heat Transfer"* Department of Mechanical Engineering, Centre for Energy Studies.
4. http://hyperphysics.phy-astr.gsu.edu/hbase/thermo/heatra.html
5. Ebling. C. P., "*Measurements and Predictions of the Heat Transfer at the Tube-Fin junction for Louvered Fin heat Exchanger*", 2003 Blacksburg, Virginia
6. Chatterjie. D, Biswas .G, Amiroudine .S., "*Numerical Investigation of forced convection heat transfer in Unsteady Flow past a row of square Cylinder*", International Journal of Heat and Fluid Flow 30(2009) ,1114-1128
7. J.P.Holman. (1976), Heat Transfer, 4nd Ed., McGraw-Hill, USA, pp 109-152. ISBN 0070295980, 9780070295988
8. Trostel R. (1956) Instationäre Wärmespannungen in einer Hohlkugel, Ingenieur-Archiv, pp 373-391. DOI: 10.1007/BF01845967
9. Verein Deutscher Ingenieure (VDI)-Wärmeatlas, 2003, Ed 14-17, Düsseldorf. ISBN 3540437126, 9783540437123
10. Han, J.-C., Dutta, S., and Ekkad, S. (2000). ''Gas Turbine Heat Transfer and Cooling Technology''. Taylor & Francis, New York.
11. Rahimi, M., Owen, I., and Mistry, J. (2003). Heat transfer between an under-expanded jet and a cylindrical surface. Int. J. Heat Mass Transfer 46, 3135–3412.