Vol.1 Issue.7, November 2013. Pgs: 149-159

ISSN (ONLINE): 2321-3051
INTERNATIONAL JOURNAL OF RESEARCH IN AERONAUTICAL AND MECHANICAL ENGINEERING

Comparative analysis of Direct Adaptive-NSGA-II & NSGA-II Optimization Algorithms

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Abstract

Engineering assessment involves many challenging objectives relies on choosing a design solution from a best set of solutions. This best set of solutions, known as the Pareto set, shows the tradeoffs that lie between the challenging objectives for diverse design solutions. Generation of this Pareto set is the major aim of multiple objective optimizations. There are various approaches to solve this kind of problem. Several approaches create solutions that cannot be applicable to combination of discrete and continuous variable parameters problems. The key objective of this paper is to demonstrate two approaches of using genetic algorithms to reduce these problems. The first approach uses a conventional non-sorted genetic algorithm to handle multiple objective optimizations, while the second method operates within the non-sorted genetic algorithm with some significant internal modifications which operates online kriging. Each approach has its strengths and weaknesses, and it is the objective of this project to compare and highlight the two approaches quantitatively as well as qualitatively. Three multiobjective optimization problems are used for the purpose of this comparison.

KEYWORDS: Optimization-based design, genetic algorithms, multiobjective optimization.

1. INTRODUCTION

A number of optimization approaches offered the solution to various non-linear problems. However, for engineers to apply optimization at their work place they need to understand the theory, the algorithm and the techniques behind these approaches. This is since practical problems may involve modifying algorithmic parameters and even scaling and adapting the existing approaches to suit the specific application criterion. Above all, the user may have to try out a number of optimization approaches to locate one that can be successfully applied.

An important obstacle in the uses of genetic algorithm (GA) based optimization approaches to engineering design problems has been the high computational cost due to the large number of simulation calls required by these approaches[1]. A general approach to decrease the computational effort for such optimization approaches when integrated with simulation models is to use some metamodeling techniques. Researchers have been

devote their effort to develop models and methods that improve the efficiency of the GAs in terms of the number of simulation calls[2–18]. Some of these approaches are based on fitness approximations in which neural network[3–5], response surface[6], Kriging[7], and radial basis function[8] approaches are used for metamodeling. Others use fitness inheritance approaches[9,10] in which the fitness of an offspring is inherited from its parents. A comprehensive review of fitness approximation and metamodeling approaches can be found in Ref. [16] and Refs. [17–19]. The fitness approximation approaches are of two types: off-line and on-line (adaptive). In off-line approaches, metamodels are created separately and prior to the start of an optimization algorithm [4,6–8,20,21]. The deficiency of the offline approaches is that it is hard to obtain both a good reliability metamodel over the entire design space and at the same time preserve a low number of simulation calls [18,20]. The adaptive approaches use a combination of metamodeling with the simulation model during the optimization procedure while adaptively improving the metamodel [2,3,5,11–14]. Most of the on-line approaches created so far are focused on single-objective optimization.

The research on how to embed metamodeling within Non-sorted genetic algorithm (NSGA-IIs) remains sparse[2,3,13]. In on-line approaches[2,3,11–13], in the initial stages of the NSGA, rough metamodels are constructed. These metamodels are then steadily improved as more simulation data become available. Another unresolved issue in the current adaptive methods is how to objectively decide when to switch to the metamodel instead of using the simulation during the optimization [11,12,16]. Usually, the switching between the actual simulation model and the corresponding metamodel is subjectively decided [13]. We can use the kriging technique, which is known to reduce a large number of simulation calls. Moreover, the fidelity of the metamodel may vary significantly during the optimization process and this can cause oscillation [16].

We use an objective criterion to determine whether a simulation model or its Kriging metamodel replacement should be used to evaluate design points. The proposed criterion is developed based on the metamodel's predicted error, which can be easily obtained as a by-product from Kriging. In the proposed approach, the Kriging metamodels for objective and constraint functions are built and adaptively improved within a NSGA-II. The approach is general and requires no additional simulation calls prior to the start of the optimization procedure to build the Kriging metamodels. These current results show that the proposed approach resolves the problem often reported in the literature, that is, the metamodel may be of low fidelity and that it may produce false optima [12,16].

2. Non-Sorted Genetic Algorithm-II (NSGA-II)

A multi-objective optimization (MOO) problem is given as follows:

$$\min_{x} f_m(x) \qquad m = 1, \ldots \ldots \ldots, M$$

Subjected to $\qquad g_j(x) \le 0 \; j = 1, \ldots, J \qquad x^{\text{lower}} \le x \le x^{\text{upper}}$

The objective $f_m(x)$ is the m^{th} element in the vector $f = (f_1, \ldots, f_M)^T$, where T refers to the transpose of the row vector. $x=(x_1, \ldots, x_N)^T$ is the design variable vector. x^{lower} and x^{upper} are the lower and upper bounds on x, respectively. The functions $g_1, \ldots, g_J$ are the inequality constraints. In this project the assumption have been made that there are trade-offs among at least two of the M objective functions. As such, the optimization problem in above equation has more than one solution.

These solutions are optimal in a Pareto sense, and the set of all such solutions forms the Pareto set or Pareto frontier. In this project, a simulation call refers to calculation of objective and constraint values together for a single design point. In the context of NSGA-II, a point is said to be "nondominated" if no other point in the current generation (or set of points under consideration) is better than that point with respect to all objectives. The set of all nondominated points in the current generation forms a nondominated set. The remaining points in the current generation form a "dominated" set. The "domination status" of a point determines whether a point is dominated or nondominated. In NSGA-II, the set of nondominated points eventually evolves to form the Pareto frontier (or an estimate of it) when NSGA-II converges.

The NSGA-II implementation is based on the standard feature of GA and includes:

- Non-dominated classification strategy
- Crowding distance evaluation to help create a good distribution of the points on the Pareto front.
- Storage of all the global non-dominated points
- Flexible termination criterion for better run control

3. DIRECT ADAPTIVE NON-SORTED GENETIC ALGORITHM (DA-NSGA-II) APPROACH

The projected approach for a multi-objective optimization technique, which optimize parameter responses created through the Kriging metamodeling technique. The main difference between this approach and a conventional NSGA-II is that, the use Kriging algorithm through which most of the optimal design points are estimated virtually by applying Kriging technique instead of the tangible simulation. In case, where the most of the objective functions has to be minimized to achieve the optimal solution.

For these types of cases the kriging technique looks for decreasing the highest entry in the diagonal of the hat matrix $x(x'x)^{-1}x'$. This has the result of reducing the highest variance of the forecasted values that proposes an improved response parameter eminence and fits multi-order deviations of the goal functions. The main initiative DA-NSGA-II is to make sure that in every generation the Pareto design points achieved by using the Kriging technique for goal and response functions remain similar as the one calculated with the simulation. For simplicity, first describe the DA-NSGA-II approach mainly for handling objective functions. After that, the brief description about how to handle constraint functions in the approach.

Kriging techniques visualize a compilation of a polynomial model in accretion with departures of the structure given by:

$$y(x) = f(x) + Z(x) \quad (1)$$

Where $y(x)$ is the objective parameter function, $f(x)$ is a polynomial function parameter of x, and $Z(x)$ is the conception of a typically distributed Gaussian random technique with mean zero, variance σ^2, and non-zero co-variance. A MOO problem is given as follows:

$$\min_x f_m(x) \qquad m = 1, \ldots, M$$

Subjected to $g_j(x) \le 0 \qquad j = 1, \ldots, J \quad x^{lower} \le x \le x^{upper}$ (2)

Where $x=(x_1, \ldots, x_n)^T$ is the shape variable vector. x^{lower} and x^{upper} are the lower and upper bounds on x, respectively. The $f(x)$ express in this associated to the polynomial model in a response surface and gives a "global" model of the design.

The co-variance matrix of $Z(x)$ is given by,

$$Cov\,[\,Z(x^i), Z(x^j)\,] = \sigma^2 R([r(x^i, x^j)]) \quad (3)$$

In R is the correspondence matrix, and $r(x^i, x^j)$ is the spatial correspondence of the parameter function between any two of the N test points x^i and x^j. R is an $m \times m$ symmetric, optimistic definite matrix with ones along the diagonal. The correspondence parameter function $r(x^i, x^j)$ is Gaussian correspondence parameter function:

$$r(x^i, x^j) = \exp\left(-\sum_{k=1}^{M} \theta_k \left| x_k^i - x_k^j \right|^2\right) \quad (4)$$

The θ_k in are the anonymous parameters employed to fit the model, M is the number of design variables, and x_k^i and x_k^j are the k^{th} components of test points x^i and x^j. In some cases, using a single correspondence parameter gives adequately high-quality results; the user can identify the use of a single correspondence parameter, or one correspondence parameter for each design.

$Z(x)$ can be written as:

$$Z(x) = \sum_{i=1}^{N} \lambda_i r(x^i, x) \quad (5)$$

Vol.1 Issue.7, November 2013. Pgs: 149-159

In any generation, except the initial population where all individuals are observed, the Kriging metamodel can be used to obtain the predicted response of individuals.
Based on these predicted response values, the domination status of individuals can be determined. To do this, the current population is partitioned into two sets: dominated and nondominated sets. Note that this partitioning is based on the Kriging
Metamodel values, that is, no simulation calls are used at this stage.

In the created design space, the minimum of minimum distance (*MD*) between every pairs of non-dominated x_{nd} and dominated x_d points and can be solved as follows:

$$MD = \min\{|| f(x_{nd}) - f(x_d)||_2\} \quad (6)$$

$x_{nd} \in \{\text{non-dominated set}\}$, $x_d \in \{\text{dominated set}\}$

Where the median is defined in the *f* space: f_1 , f_2 , . . . , f_M. MD is then estimated along each objective parameter function axis to acquire

$$MDf_m,\ m=1, \ldots, M \quad (7)$$

The Mean squared error of an overlooked point x^* for objective parameter function f_m is

$$\text{Mean square error (MSE)} = S_m(x^*). \quad (8)$$

So the predicted error (E) of x^* as

$$E_m(x^*) = S_m(x^*)/2, \quad m = 1, \ldots, M \quad (9)$$

For objective functions, the criterion for the DA-NSGA-II is devised as follows:

(1) Since $E_m(x^*)$estimates a deviation from the actual response, the sum of E values for the m^{th} objective function of any pair of nondominated point x_{nd} and dominated point x_d, $E_m(x_{nd})+E_m(x_d)$, is the possible error when calculating the distance between the two points by that objective. In the worst case, this sum should be less than MDf_m, for all objectives, m=1, . . . , M; otherwise, the predicted error of any pair of x_{nd} and x_d may change the domination status. For instance (whereby both objective functions being minimized and Point *a* dominates Point *i*), if $Em(a)+E_m(i) \le MDf_m$ for all m=1,2, then the domination status of Points *a* and *i* should not change. In fact, this criterion should hold between any x_{nd} from the nondominated set and any x_d from the dominated set.

(2) Mathematically, $E_m(x_{nd}) + E_m(x_d) \le MDf_m$ implies that

$$\text{Max}(E_m(x_{nd}),E_m(x_d)) \le MDf_m/2$$

(3) In the worst case, if $2\times E_m(x) \le MDf_m$ and thus $s_m(x) \le MDf_m$ is true for all m=1, . . . ,M and for design point x, then the domination status of point x should not change in the current population.

In short, if for any test point x the following holds:

$$S_m(x) \le MDf_m \quad (10)$$

for all $m=1, \ldots, M$, then the predicted response values by Kriging technique for x will be considered as "good quality" values. On the other hand, the absolute value of $g_j(x)$ offers to mitigate for the predicted error along g_j dimension. If this absolute value is superior to $2\times s_j(x^*)$, then the predicted restraint value has a very slight chance i.e., less than 3% to adapt feasibility of the design x^*. Thus, if for any test point x the following condition holds:

$$2\times s_j(x^*) + g_j(x) \le 0 \quad (11)$$

After putting together, Eq. (10) with Eq. (11), as shown in Eq. (12):

$$\begin{aligned} &S_m(x) \le MDf_m && m=1 \ldots M \\ &2\times s_j(x^*) + g_j(x) \le 0 && j=1,\ldots..,J \end{aligned} \quad (12)$$

The designs in the recent population can be separated into two groups. In the first group that gratifies Eq. (12), the estimated responses will not modify the domination condition of the designs and also these designs are predicted to be feasible and therefore their estimated response values can be established. For the designs in this first group, no simulation is necessary. Design solutions in the second group do not satisfy Eq. (12). As such, the solutions in this second group are estimated by the simulation to acquire their objective/constraint values, as shown in the "Direct (online) Kriging assisted fitness assessment". The optimum solution is then checked with the algorithm stopping criterion set by the goal driven non sorted genetic algorithm-II. Finally when the criteria satisfy then the optimum solution will be available.

4. STEPS FOR DA-NSGA-II TECHNIQUE

Direct (online) Adaptive Non Sorted Genetic Algorithm – II (DA-NSGA-II) Steps:

1. **First Population of NSGA-II**

The initial population of NSGA-II is used for the Kriging raw metamodel construction.

2. **Kriging Generation**

A Kriging response surface is created for each output, based on the first population and then improved during simulation with the addition of new design points.

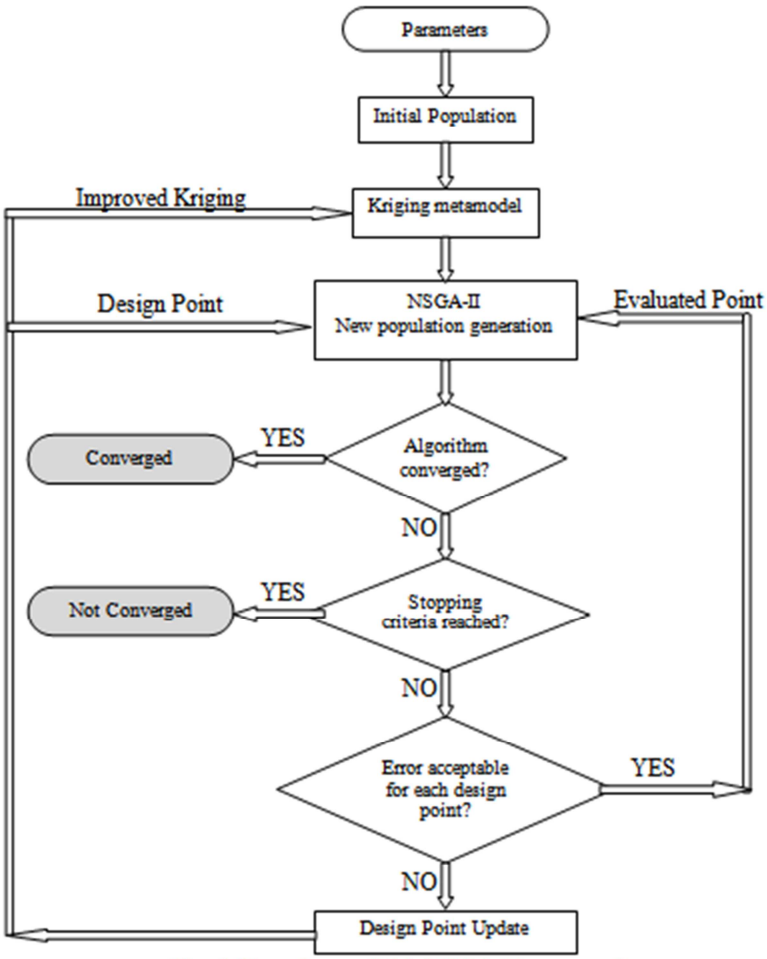

Fig. 1 Flowchart of DA-NSGA-II approach

3. **NSGA-II Algorithm**

NSGA-II is run, using the Kriging as an evaluator. After the first iteration, each population is run when it reaches the number of samples.

4. **Convergence Validation**

After the NSGA-II algorithm is run, the optimization is validated for convergence.

- **Optimization Converged**

NSGA-II converges when the maximum allowable Pareto percentage has been reached. When this happens, the process is stopped.

- **Optimization Not Converged**

If the optimization is not converged, the process continues to the next step.

5. **Stopping Criteria Validation**

If the optimization has not converged, it is validated for fulfilment of stopping criteria.

- **Stopping Criteria Met**

When the maximum number of iterations has been reached, the process is stopped.

- **Stopping Criteria Not Met**

If the stopping criteria have not been met, the process continues to the next step.

6. **Error Check**

If stopping criteria have not been met, the Kriging error predictor is checked for each point.

- **Error Acceptable**

Each point is validated for error. If the error for a given point is acceptable, the approximated point is included in the next population to be run through the NSGA-II algorithm (return to Step 3).

- **Error Not Acceptable**

If the error is not acceptable, the points are promoted as design points. The new design points are used to improve the Kriging (return to Step 2) and are included in the next population to be run through the NSGA-II algorithm (return to Step 3).

7. **Conclusion**

Steps 2 through 6 are repeated in sequence until the optimization has converged or the stopping criteria have been met. When either of these things occurs, the final population is evaluated and the optimization concludes.

5. THE IMPLEMENTATION OF THE APPROACH

In this section, we first describe the test problems used to compare the performance of NSGA-II with DA-NSGA-II. For NSGA-II, we have identical parameter settings as suggested in K. Deb, 2001. For DA-NSGA-II, we have chosen a reasonable set of values and have not made any effort in finding the best parameter setting. We leave this task for a future study.

Problem 1: OSY Problem Definition

We applied the NSGA-II which was described in K. Deb, 2001 and DA-NSGA-II to this problem that has two objective functions, six constraints, and six input parameters as shown below:

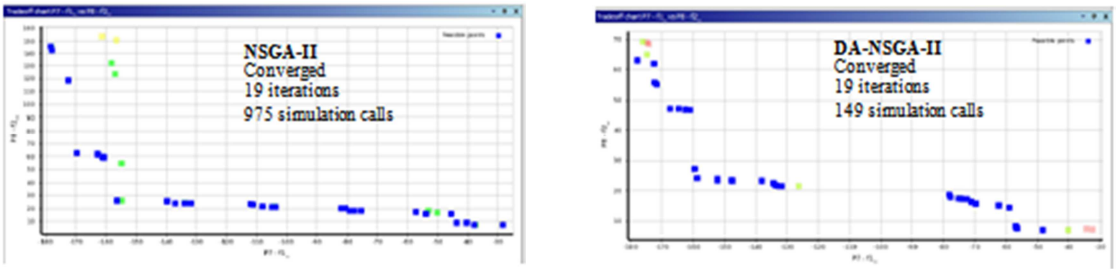

Fig. 2 NSGA-II Vs DA-NSGA-II comparisons based on Number of simulation calls (*NumSimCall*) for OSY

Figure 2 shows a typical set of Pareto optimal solutions as obtained from one of the 19 iterations of the NSGA-II and DA-NSGA-II. As shown in Figure 2, a NSGA-II run with the least number of simulation calls (i.e., 975 in run 19) requires more simulation calls than a DA-NSGA-II run with the maximum number of simulation

calls (i.e., 149). The results show that for OSY problem, the *NumSimCall* has been reduced by more than 84% (824 fewer points) using the proposed DA-NSGA-II compared to the NSGA-II, Because the DA-NSGA-II method required 826 fewer points, the optimization process was much faster.

Problem 2:

In this section, the numerical problem in B. Wilson, 2001 [21] is presented to demonstrate further applicability of the DA-NSGA-II. The analytic function we will examine for the Problem has two input parameters and four output parameters (with two objectives and two constraints). The obtained Pareto solutions using the NSGA-II and DA-NSGA-II are shown in Figure 3.

The definition of the problem is as follows:

Minimize

$$f_1(x_1,x_2) = (x_1 + x_2 - 7.5)^2 + (x_2 - x_1 + 3)^2 / 4$$
$$f_2(x_1,x_2) = (x_1 - 1)^2 / 4 + 2$$

With Subjected to

$$g_1(x_1,x_2) = (x_1 - 2)^3 / 2 + x_2 - 2.5 \leq 0$$
$$g_1(x_1,x_2) = x_1 + x_2 - 8(x_2 - x_1 + 0.65)^2 - 3.85 < 0$$

Where $0 \leq x_1 \leq 5$ and

$0 \leq x_2 \leq 3$

Figure 3 shows the *NumSimCall* (number of simulation calls) for 20 iterations. As shown in Figure 3, Pareto front found by the DA-NSGA-II optimization is exactly the same one that was found by the NSGA-II optimization; the results for each method are identical. The difference is in the number of design points needed to reach this result. Whereas NSGA-II had to run 446 design points, DA-NSGA-II only had to run 139 design points. The results show that for numerical problem, the *NumSimCall* has been reduced by more than 69% (307 fewer points) using the proposed DA-NSGA-II compared to the NSGA-II, Because the DA-NSGA-II method required 307 fewer points, the optimization process was much faster.

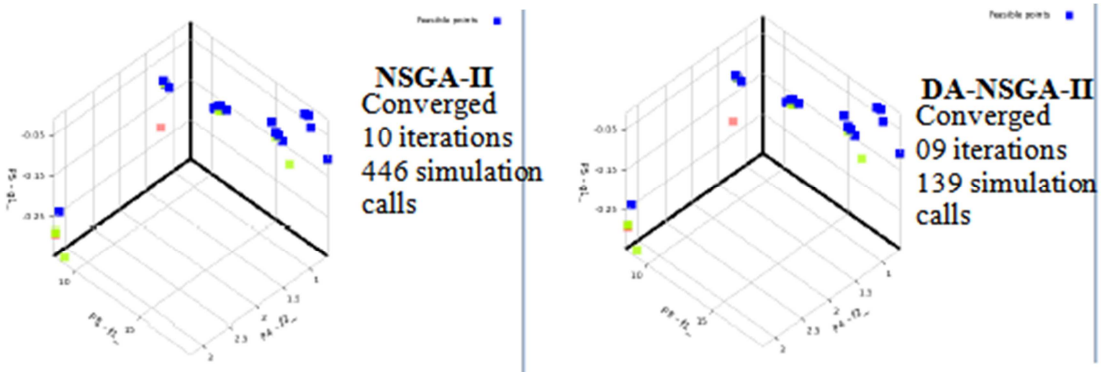

Fig. 3 NSGA-II Vs DA-NSGA-II

Problem 3:

This problem is a relative simple problem taken from Kirsch (1981) [37] with some modifications that involves the minimization of volume and stress of a *two-bar truss*. It is comprised of two stationary pinned joints, A and B, where each one is connected to one of the two bars in the truss. The two bars are pinned where the join one another at joint C, and a 100 kN force acts directly downward at that point. The cross-sectional areas of the two bars are represented as *x1* and *x2*, the cross-sectional areas of trusses AC and BC respectively. Finally, *y* represents the perpendicular distance from the line AB that contains the two-pinned base joints to the connection of the bars where the force acts (joint C). The *two-bar truss* is shown below.

Vol.1 Issue.7, November 2013. Pgs: 149-159

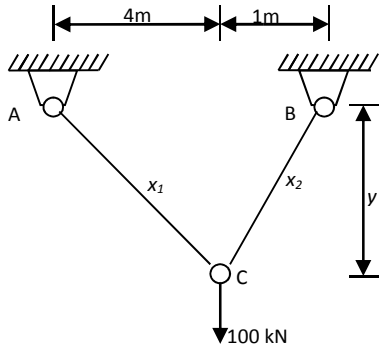

Fig. 4 Two-Bar Truss

The problem has been modified into a two-objective problem in order to show the non-inferior Pareto set clearly in two dimensions. The stresses in AC and BC should not exceed 100,000 kPa and the total volume of material should not exceed 0.1 m^3. Hence, in order to generate Pareto optimal solutions in a reasonable range, objective constraints are imposed. The problem formulation is shown below:

Minimize: $f_{volume} = x_1 (16 + y^2)^{0.5} + x_2 (1+y^2)^{0.5}$

$$f_{stress,AC} = \frac{20(16 + y^2)^{0.5}}{yx_1}$$

Subjected to: $f_{volume} \leq 0.1$

$f_{stress,AC} \leq 100000$

$f_{Stress,BC} \leq 100000$

Where, $1 \leq y \leq 3$

$0 \leq x_1, x_2 \leq 10000$

$$f_{Stress,BC} = \frac{80(1 + y^2)^{0.5}}{yx_2}$$

For this problem NSGA-II had to run 570 design points, DA-NSGA-II only had to run 200 design points. The results show that for two bar truss problem, the *NumSimCall* has been reduced by more than 65% using the proposed DA-NSGA-II compared to the NSGA-II.

6. Comparison of NSGA-II and DA-NSGA-II

The obtained results for these three problems show that the number of simulation calls (*NumSimCall*) used in the DA-NSGA-II is significantly fewer than the NSGA-II, while the obtained Pareto solution for NSGA-II method is comparable. Furthermore, as shown in Table 1, the DA-NSGA-II has smaller STD of the *NumSimCall* (based on 20 iteration runs) than the NSGA-II, which indicates that compared to the NSGA-II and the DA-NSGA-II has a more stable performance on the reduction of the *NumSimCall*.

Table 1 Statistics for the *NumSimCall*

S. No	Problem	NSGA-II		DA-NSGA-II	
		Mean	STD	Mean	STD
1	All three problems	570	225.89	149	26.71

Based on the data in Table 2, the reduction of the *NumSimCall* for each problem is calculated based on the mean and STD value. This calculation performing for the DA-NSGA-II over the NSGA-II, is also shown in Table 2.

Table 2 Reduction in the *NumSimCall* & Time required

S. No	**Problem**	**Reduction in *NumSimCall*** $1-(\frac{DA-NSGA-II}{NSGA-II})$	**Time (Min)**	
			NSGA-II	***DA-NSGA-II***

1	OSY Problem	84%	25	10
2	Problem 2	69%	12	06
3	Problem 3 (Two bar truss)	65%	10	05

As shown in Table 2, on the average, the proposed DA-NSGA-II can save about 72.6% in the *NumSimCall* and 43% of time over the NSGA-II. It is observed that the DA-NSGA-II outperforms the NSGA-II and is more stable than the NSGA-II, in terms of the number of simulation calls, for these three standard problems.

7. SUMMARY

An improved multi-objective design optimization approach called DA-NSGA-II is presented in this paper. In the proposed approach, the online kriging metamodel is embedded within a conventional NSGA-II. However, compared to the NSGA-II, DA-NSGA-II reduces the number of simulation calls by applying a new objective criterion in the response prediction from the online kriging metamodel and evaluating some individuals in the population by the kriging metamodel instead of the simulation and the performance of the DA-NSGA-II in the reduction of simulation calls is more stable than NSGA-II. For future work, we anticipate that the simulation of some subsystems will be very expensive (it takes several hours or even days for one simulation run) or the entire system can be very complex, and may have numerous subsystems. As a result, only a very limited number of simulations may be available. The Design of Experiments (DOE) and metamodeling methods could be suitable for the decomposed (or decentralized) multidisciplinary design environment. Thus an efficient decomposition based approximation approach with a decomposed formulation becomes critical.

Based on these investigations, it can be expected that using the proposed metamodel assisted NSGA as the optimizer with improved objective criteria can improve the efficiency of robust optimization problems where the accuracy of involved metamodels becomes more important.

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