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Finite Element Method Dispersion Curves of Sandwich Skin

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Abstract

In this paper we aim investigating the Finite Element Method (FEM) and Two Dimensional Fast Fourier Transform (2D-FFT) regarding DISPERSE program to reconstruct dispersion curves of Sandwich Skin. The resolution of the equation based on the finite element method permits to obtain the displacements which are, after that, post processed by the 2D-FFT. To minimize the inaccuracy of the proposed method, parameters must be optimized such as: frequency excitation, model element type, spatial and temporal steps.
The investigation is carried out using Abacus CAE. This study allows knowing the optimal parameters to generate dispersion curves depending on the geometry and the materiel of the investigated plate.

Keywords: Sandwich Skin; Dispersion curves; Finite Element Method; Fast Fourier Transform.

1. Introduction

Non Destructive Testing (NDT) is a set of methods that can be used to characterize the state of integrity of structures or materials, without degrading them, either during production, in use, or in the maintenance.

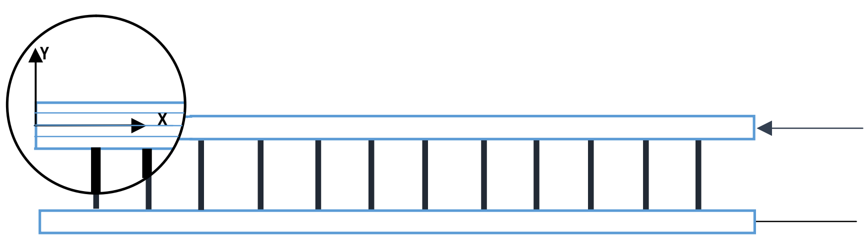

Figure 1: Sandwich structure

A sandwich structure is made from two skins bonded to a thick core as presented in figure 1. They are used especially in aeronautic industries; their mechanical proprieties are improved by the association of fibres and matrix proprieties. However, they are heterogeneous which leads to their weakness and facilitates the appearance of internal and external damages such as: fibre breakage, matrix cracking, through-thickness hole, local de lamination to control sandwich structures we need to choose a specific frequency. To do so, it is important to know the dispersion curves of tested plate: graphs indicating the propagating modes in terms of frequency.

Plotting dispersion curves still more challenging task especially for complex structures. Many researchers have contributed on in plotting dispersion curves. They suggested many techniques such as: Iterative methods: Bisection, Newton Raphson, Semi Analytical Finite Element (SAFE), Pencil matrix, Transfer matrix, Global matrix, and spectral method.

Mozer [1] used FEM to represent accurately ultrasonic waves, the authors compare the analytical solution of guided waves with numerical solution, and they excited high frequencies by triangular signal and applied 2D-FFT to in plane displacement. Mazzotti [2], studied orthotropic thin-walled. The author extracts the dispersion curves: phase velocity, energy velocity and attenuation using SAFE. This technique was also explained in Rose's book [3]. Stefan [4] proposed a method to obtain numerical dispersion curves of propagating modes by solving eigenvalue problem. The author studied square tube pipe and composite structures. Schopfer [5] introduced a method permitting extraction of dispersion curves from laser vibrometer measurement; they used Fourier Transform into the wave number domain and then applied the Matrix Pencil technique to extract the wave number-dependent frequencies. Hernando [6] extended the Spectral Method to complicated structures: orthotropic cylinder and plate and also for multi-layered systems with both solid and fluid layers. Barra [7] used a numerical resolution of the dispersion equations of orthotropic plate by searching the zeros in the (f, vp) plane using the Newton-Raphson method. Harsh [8] presented theoretical and numerical approaches the Waveguide Finite Element (WFE) method to determine the phase and group velocities for aluminium plate woven composite laminate, a sandwich panel with woven composite face sheets and an aluminium honeycomb core. Mark [9] studied experimentally the dispersion curves in aluminium and composite plates with a Q-switched excitation laser and continuous Photo-EMF detector laser interferometer. Fendzi [10] used an approach based on a Finite Element Modelling for the calculation of the dispersion curves. The approach consists of calculating periodic solutions using a Fourier / Floquet transform by assuming the existence periodic cells in the plate. Ameneh [11] developed the dispersion curves calculation in multilayered composite-metal plates using the Transfer Matrix Method. Yago [12] obtained by 2D-FFT the experimental dispersion curves using an optical vibrometer, the author compared the obtained curves with those calculated numerically by Finite Element Model using PZFlex. Lasova [13] used two approaches to calculate group velocities of zero-order Lamb wave modes in aluminium: Two-dimensional Fast Fourier Transform (2D-FFT) and methods of time-frequency processing. Hora [14] used Fourier transform to go from time to the frequency domain. Weimin [15] analyzed the dispersion curves of isotropic plate by Two Dimensional Fourier Transform method to identify the propagating modes and to determine the thickness and bulk velocities, and the elastic constants. Pablo [16] Obtained numerically lamb wave modes and dispersion curves in plates using COMSOL Multiphysics, after that data are post processed using 2D FFT. Eduardo [17] presented the phase velocity method in composite plates to evaluate Carbon Fibre Reinforced Polymer (CFRP) elastic properties. Quiroga [18] presented a method to calculate dispersion curves for homogeneous plate's subject using SAFE and the Effective Elastic Constants. Becker [19] used three methodologies to calculate the dispersion curves. Based on 3 types of element, dispersion curves were obtained by FEM. Sauvik [20] proposed two-dimensional semi-analytical model based on global matrix technique to study the characteristics of lamb wave propagation in a honeycomb composite sandwich. The author used also Wavelet Transform to obtain the group velocities of the propagating modes. Pablo [21] used the Floquet-Bloch theory to compute dispersion curves for unit cell, and to estimate the elastic parameters of a real steel slab. The same theory was used by Biyu Tian [22]. Karen [23] used Morlet Wavelet Transform to extract Dispersion curves from a single broadband signal containing several modes in aluminium plate. Lina [24] measured the group velocity based on the spectrum decomposition approach. The simulated and experimental signals of Lamb wave propagating in a 2 mm thickness aluminium plate. Also, the author [25] presented the zero-crossing technique for measurement of phase velocity of Lamb waves (the A0 and S0 modes) using modelled dispersed signals and experimental signals obtained for an aluminium plate having thickness of 2 mm. Florian [26] proposed a new spectral-method algorithm to study wave propagation in cylindrically layered fluid and elastic structures by Chebyshev points in the radial direction, also for multilayered cylindrical structures. Pablo [27] proposed numerical method based on the MASW (Multichannel Analysis of Surface Waves) to obtain dispersion curves of steel plate using COMSOL software. Harb, [28] presented , fully non-contact, hybrid system which encompasses an Air-Coupled Transducer (ACT) and a Laser Doppler Vibrometer (LDV) for profiling A0 Lamb wave dispersion of an isotropic aluminium plate. Packo [29] presented a method for dispersion curve calculation and analysis of numerical models for guided waves. The proposed approach utilizes the wave equation and through-thickness-only discretization of anisotropic, layered plates to obtain the Lamb wave characteristics. Farhang [30] proposed an alternative method which extracts the solution of the frequency equation in the form of dispersion curves from the three-dimensional illustration of the frequency equation. For this purpose, a three-dimensional representation of the real roots of the frequency equation is first plotted. The dispersion curves, which are the numerical solutions of the frequency equation, are then obtained by a suitable cut in the velocity–frequency plane.

The methods cited above used to resolve the lamb wave propagation in anisotropic plate still difficult and require more programming skills, so numerical simulation is the best way to find out all the modes. The objective of this paper is to investigate the FEM parameters influencing on dispersion curves precision. In the first part a mathematical model is given to find out the equation governing the lamb waves' propagation in orthotropic plate. In the second part, a numerical model based on FEM explicit Abaqus is carried out to extract dispersion curves. In the last part, the obtained dispersion curves are compared with DISPERSE Program.

2. Equation of lamb waves propagation in unidirectional lamina

The characteristic equations of the symmetrical and anti-symmetric waves are [31]:

$$\begin{cases} D_{11}D_{23}cotan\left(\frac{k\alpha_1 h}{2}\right) - D_{13}D_{21}cotan\left(\frac{k\alpha_3 h}{2}\right) = 0 \\ D_{11}D_{23}tan\left(\frac{k\alpha_1 h}{2}\right) - D_{13}D_{21}tan\left(\frac{k\alpha_3 h}{2}\right) = 0 \end{cases}$$

With: D_r Coefficients calculated by following equations: $\begin{cases} D_{1r} = C_{13} + C_{33}\alpha_r W_r \\ D_{2r} = C_{55}(\alpha_r + W_r) \end{cases}$

$W_r = \frac{\rho c^2 - C_{11} - C_{55}\alpha_r^2}{(C_{13}+C_{55})\alpha_r}$. For r $\in$ {1, 2, 3, and 4}, with:

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$$\begin{cases} \alpha_1 = -\alpha_2 = \dfrac{-B - \sqrt{B^2 - 4AC}}{2A} \\ \alpha_3 = -\alpha_4 = \dfrac{-B + \sqrt{B^2 - 4AC}}{2A} \\ \alpha_5 = -\alpha_6 = \sqrt{\dfrac{\rho c^2 - C_{66}}{C_{44}}} \end{cases}$$

Coefficients A, B and C are calculated by following equations:

$$\begin{cases} A = C_{33}C_{55} \\ B = C_{33}(C_{11} - \rho c^2) + C_{55}(C_{55} - \rho c^2) - (C_{13} + C_{55})^2 \\ C = (C_{11} - \rho c^2)(C_{55} - \rho c^2) \end{cases}$$

Where h is the skin thickness, k is the wavenumber corresponding to the X direction, c is the phase velocity. C_{11}, C_{13}, C_{33}, C_{44}, C_{55}, C_{66} are the elastic constants, and ρ is the skin density.

3. Numerical model

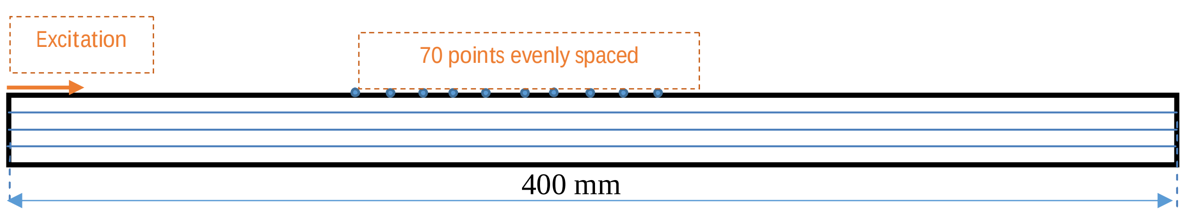

Figure 2: Model geometry of skin sandwich

Table 1: Elastic coefficients C_{ij} and density ρ of the materiel

ρ (kg/m^3)	C_{11}(GPa)	$C_{22} = C_{33}$	$C_{12} = C_{13} = C_{23}$	$C_{55} = C_{66}$
1530	56,9	14,7	9,76	4,16

3.1 Meshing and time sampling

To satisfy an accurate solution, the model has been meshed using the equation 6, and the time step is calculated by equation 7:

$$Max(\Delta x, \Delta y) < \frac{\lambda_{min}}{10} \quad (2)$$

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$$\Delta t < 0.7\frac{Max(\Delta x,\Delta y)}{V_L} \tag{3}$$

3.2 Boundary conditions

The skin left is excited with a displacement boundary condition in the X direction. The figure 3 shows the time function of in plane displacement.

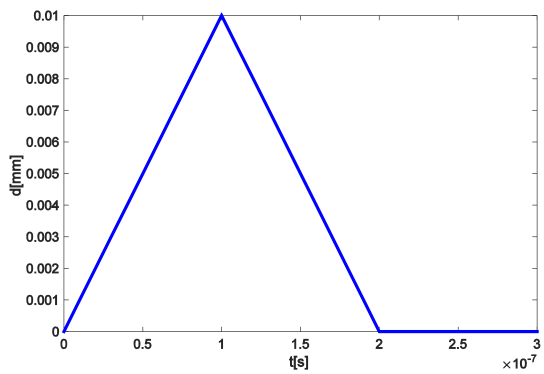

Figure 3: Multimode excitation by triangular in plane displacement

4. Extraction Method

4.1 Method

To plot dispersion curves from temporal-displacement data S(t,x) extracted from Abaqus software, we start first of all by applying 2D FFT, the obtained frequency-wave number data S(f,k) based on Fourier Diamond (see figure 4) are then plotted over analytical dispersion curves. Finally, the error is evaluated by comparison.

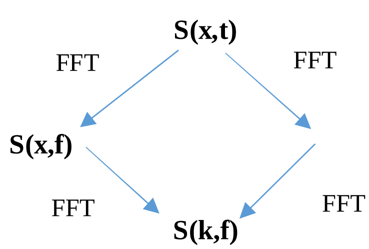

Figure 4: Fourier Diamond

4.2 2D-FFT

Lamb waves' propagation is sinusoidal in the frequency and spatial domains. For that reason, a temporal Fourier Transform is applied to go from the time to frequency domain, after that, a spatial Fourier Transform is computed to obtain the frequency wave-number domain, see [32]. In practice, carrying out spatial Fourier methods to data obtained experimentally or numerically requires applying a 2D-FFT, using:

$$H(k,f) = \iint_{-\infty}^{+\infty} u(x,t) e^{-i(kx+\omega t)}\, dxdt \tag{4}$$

5 Results

The temporal-displacement data S(x,t) is showed in figure 5. The 70 points evenly spaced are picked up from the top surface of the modeled skin. The figure also shows the incident and the reflected modes.

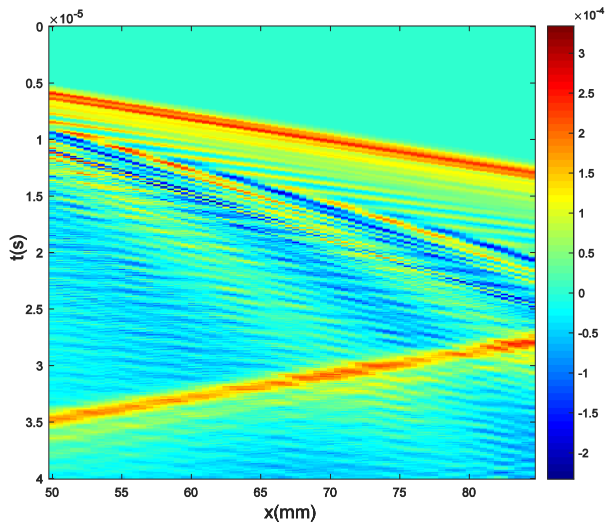

Figure 5: Wavefields of propagating modes

The figure 6 illustrates the frequency-wave number data S(f,k) obtained by 2D-FFT. It shows the 3 first propagating modes which are respectively: A_0, S_0, and A_1.

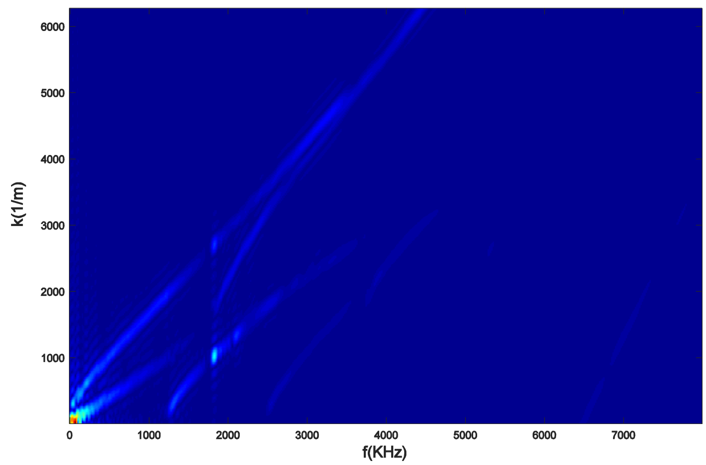

Figure 6:FEM Dispersion curves of skin sandwich

The numerical dispersion curves obtained by FEM are plotted over DISPERSE curves as shown in fig. 7.

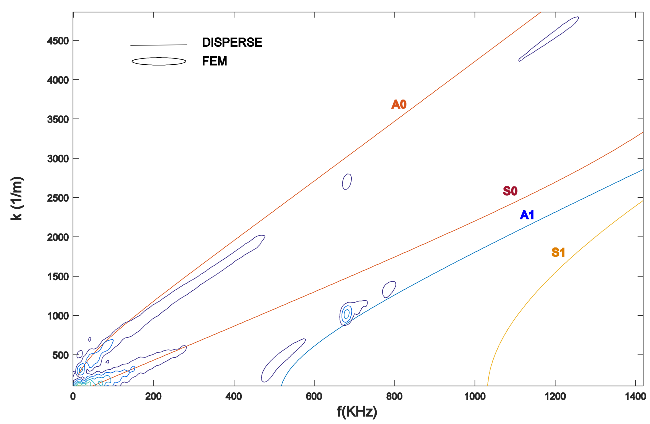

Figure 7: Comparison between FEM and DISPERSE dispersion curves.

DISPERSE program, developed by Imperial College NDT Laboratory [33], permits to create dispersion curves, it is based on global matrix method, the displacements and stresses are described in a material layer matrix. By satisfying the boundary conditions at each interface, the individual layer matrices are assembled.

The results showed in figure 7 are accurate enough to choose an optimal frequency for experimental work. However, there are inaccuracies due to the approximated model. Moreover, the figure indicates that the error increases when the frequency grows up. This is called the thickness frequency problem.

In practical work, the controller chooses a frequency where there are less propagating modes. In the case of skin sandwich, as presented in figure 7, the range of frequency to be chosen should be less than 500 KHz in order to prevent the third mode propagation. At this frequency the error is less than 10%.

6 Conclusion:

In this paper, we have studied the numerical model describing the propagation of lamb waves within a sandwich skin. Simulations based on Finite Element Methodwere carried out using ABAQUS Software in order to investigate the FEM parameters influencing the dispersion curve extraction. The resultscompared with DISPERSE program. It showed that the obtained dispersion curves can be used as they give accurate information with an error less than 10%.

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