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RADAR BASED NONLINEAR ORBITAL TRAJECTORY ESTIMATION

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Abstract

The estimation of space object trajectories in the framework of space situational awareness and collision warning is based on the availability of highly accurate information about the orbiting objects. Such a task is critical and radar tracking of space objects requires robust filtering methods that are able to deal not only with the uncertainties of the vehicle dynamics but also with environmental disturbances and instrumental inaccuracies related to the data acquisition systems. One of the main tasks related to radar tracking of space objects is the nonlinear filtering of trajectory data. To address the limitations due to the high computational load required by current nonlinear filtering methods, the present paper proposes a nonlinear state estimation method based on the pseudolinearization of the system model, thus resulting in local models that are exactly equivalent to the original nonlinear model, therefore allowing accurate estimation to be achieved in radar tracking of space objects. Besides, the proposed method enables to estimate the unknown statistical characteristics of the environmental disturbances and instrumental inaccuracies without significant increase in the processing time. The proposed method has been successfully validated on low-earth-orbit applications in a realistic simulation framework.

Keywords: Nonlinear Filtering, Pseudolinearization, Low-Earth-Orbit Estimation

1. Introduction

This paper considers the problem of determining a state vector from a set of noisy measurements, applying the Kalman filter to the nonlinear system.

In nature, most physical systems are inherently nonlinear so the nonlinear state estimation plays an important role in a wide frame of applications from target tracking, navigation for the aerospace vehicle, multi-sensor data fusion to different areas such as estimation of structural macroeconomic model. That is why it is an important topic in modern control theory and control system engineering.

Estimation in nonlinear systems is extremely difficult and huge efforts have been made in the past years to obtain results and data with more accuracy and stability, still is space for big improvements.

This work was realized with the intention of contributing to an improvement on estimation problem of nonlinear systems.

2. Linear Kalman Filter

Under linearity and Gaussian conditions on the systems dynamics, the general filtering problem particularizes to the Kalman filter, which is the most widely used method for tracking and estimation due to its simplicity, optimality, tractability and robustness.

The Kalman filter can be described as an optimal recursive data processing algorithm, ie, an optimal linear estimator [1], when applied to discrete time, finite dimensional time-varying systems evaluates the state estimate and simultaneously minimizes the mean-square error. It dynamics results from the consecutive cycles of prediction and filtering. The dynamics of these cycles is derived and interpreted in the framework of Gaussian probability density functions (Bayesian solution). Under additional conditions on the system dynamics, the Kalman filter dynamics converges to a steady-state filter and the steady-state gain is derived.

The filter objective is to calculate an estimate of the system state based on measurement vector; it has a predictor-corrector structure, which allows to estimate the future state based on past observations.

Given a discrete-time process governed by linear stochastic equations allows the Kalman filter estimate the state, $x \in R^n$ of a system at a time k by assuming that the state evolved from *a priori* state at time $(k-1)$ according to the equation [2]:

$$x_k = A x_{k-1} + B u_k + w_k \qquad (1)$$

where, x_k is the state vector at time k that contains the parameters of interest for the system; A is a matrix $(n \times n)$ that relates the state at a previous time step $(k-1)$ to the state at the actual step k ; B is a matrix $(n \times l)$ that relates the optimal control input $u \in R^l$ to the state x ; w_k is the process noise terms for each parameter in the state vector and is assumed to be independent, zero mean Gaussian white noise with covariance Q_k, defined as $Q_k = E\left(w_k w_k^T\right)$; E is the expected value (first statistical moment).

The measurements, $z \in R^m$ of the system are performed according to:

$$z_k = H x_k + v_k \qquad (2)$$

where, z_k is the measurement vector at time k ; H is a matrix $(m \times n)$ that relates the state to the measurements; v_k is the measurement noise terms for each observation in the measurement vector. Like the process noise, is assumed to be independent, zero mean Gaussian white noise with covariance R, given by $R_k = E\left(v_k v_k^T\right)$.

The estimation process of Kalman filter is based in a feedback control method, where the system state is estimate at a given time and then coupled with the feedback obtained through the noisy measurements.

The estimation algorithm of Kalman filter is divided into two groups:

1) time update equations, also known as predictor equations, since they are responsible to project *a priori* estimate for the state in the forward time step;

2) measurements update equations are responsible to integrate the feedback in the estimate, this means, a new measurement will be incorporated into the *a priori* estimate to achieve an improved *a posteriori* estimate, that is why this group of equations is also known as corrector equations.

The standard Kalman filter for the prediction group are given by [2][3]:

$$\hat{x}_k^- = A\hat{x}_{k-1} + Bu_k \qquad (3)$$

$$P_k^- = AP_{k-1}A^T + Q \qquad (4)$$

being, $\hat{x}_k^-$ the *a priori* state estimate; $\hat{x}_{k-1}$ the state estimate at time step $(k-1)$; P_k^- the *a priori* covariance vector; P_{k-1} the covariance vector at time step $(k-1)$;

The measurement update equations are:

$$K_k = P_k^- H^T \left(HP_k^- H^T + R\right)^{-1} \qquad (5)$$

$$\hat{x}_k = \hat{x}_k^- + K_k\left(z_k - H\hat{x}_k^-\right) \qquad (6)$$

$$P_k = (I - K_k H)P_k^- \qquad (7)$$

where, K_k is the Kalman gain at step k ; $\hat{x}_k$ is the *a posteriori* state estimate, that means it takes into account the measurement z_k ; I is the identity matrix.

The difference $z_k - H\hat{x}_k^-$ is known as residual and is interpret as the variance between the predicted measurement $H\hat{x}_k^-$ and the actual measurement z_k .

3. Extended Kalman Filter (EKF)

The extended Kalman filter is probably the most widely used estimation algorithm for nonlinear systems. It applies the Kalman filter to nonlinear systems by linearizing the models, that means, the EKF has the ability to linearize the current mean and covariance [2][3].

Given a state dynamics of a general nonlinear time-varying system defined as $x \in R^n$:

$$x_k = f(x_{k-1}, u_k) + w_{k-1} \qquad (8)$$

and being the measurement vector $z \in R^m$:

$$z_k = h(x_k) + v_k \qquad (9)$$

where, k is the time index; f is non-linear function that relates the state at time step $(k-1)$ with the current step k ; h is a non-linear equation that relates the state x_k with the measurement z_k .

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Due to the nonlinearity of the system, the EKF is implemented on a set of approximations performed by the linearization process.

The linearized model is given by [2][3]:

$$x_k = \tilde{x}_k + A_k\left(x_{k-1} - \hat{x}_{k-1}\right) + w_{k-1} \tag{10}$$

$$z_k = \tilde{z}_k + H_k\left(x_k - \tilde{x}_k\right) + v_k \tag{11}$$

where x_k, z_k are the actual state and measurement vectors; $\tilde{x}_k, \tilde{z}_k$ are the approximate state and measurement vectors; $\hat{x}_k$ is *a posteriori* estimate state at step k; A_k is the Jacobian matrix of partial derivatives of f with respect to x and is defined as:

$$A_k = \left.\frac{\partial f\left(x, u\right)}{\partial x}\right|_{\substack{x=\hat{x}_k \\ u=u_k}} \tag{12}$$

H_k is the Jacobian matrix of partial derivatives of h with respect with respect to x defined as:

$$H_k = \left.\frac{\partial h\left(x\right)}{\partial x}\right|_{x=\tilde{x}_k} \tag{13}$$

The linearization is a very important part of this process because allows the filter to get the best benefit from the available *a priori* information.

As happen with the discrete Kalman filter, the extended algorithm is also divided in two stages:

i) Time update equations:

$$\hat{x}_k^- = f\left(\hat{x}_{k-1}, u_k\right) \tag{14}$$

$$P_k^- = A_k P_{k-1} A_k^T + w_k Q_{k-1} w_k^T \tag{15}$$

These set of equations are responsible for the predictions update.

ii) Measurement update equations:

$$K_k = P_k^- H_k^T \left(H_k P_k^- H_k^T + V_k R_k V_k^T\right)^{-1} \tag{16}$$

$$\hat{x}_k = \hat{x}_k^- + K_k\left(z_k - h\left(\hat{x}_k^-\right)\right) \tag{17}$$

$$P_k = \left(I - K_k H_k\right) P_k^- \tag{18}$$

where, Q_k is the noise covariance at time step k defined by $Q_k = E\left(w_k w_k^T\right)$ being E the k^{th} statistical moment of a continuous variable; R_k is the measurement covariance at time step k defined by $R_k = E\left(v_k v_k^T\right)$.

Exploiting the assumption that all transformations are quasi-linear, the EKF simply linearizes all nonlinear transformations and substitutes Jacobian matrices for the linear transformations in the Kalman filter equations.

Although the EKF maintains the computationally efficient recursive update form of the Kalman filter, it suffers several serious limitations.

The principal limitation is that the distributions or in the continuous case, the densities of the variables are no longer normal after their own nonlinear transformations. This leads to EKF to act as an *ad hoc* estimator that only approximates the optimality of Bayes rule by linearization.

4. Unscented Kalman Filter (UKF)

To overcome the limitations presented in EKF, it was developed a new method that has the ability to propagate the mean and covariance information through nonlinear transformations [4,5].

This method is the unscented transformation, although is resembling to Monte Carlo type methods, there is a fundamental difference: the samples are not drawn at random but rather according to a specific, determinist algorithm. In this case, the problems of a statistical convergence are not an issue, so a high order information about the distribution can be calculated using a very small number of points, known as sigma points.

The UKF was developed with the assumption that approximating a Gaussian distribution is easier than approximating a nonlinear transformation [4].

4.1 Estimations

In a simple way, the unscented Kalman filter follows the steps:

i) create a sigma points;
ii) run these points through a process model;
iii) compute transformed mean and covariance;
iv) calculate the Kalman gain;
v) update the state estimate with the current measurements.

To achieve this process, the filter starts with the predicted mean and covariance of the state:

$$\hat{x}_{k_0} = E(\hat{x}_0) \quad (19)$$

$$P_{x_0} = E\left(\left(x_{k_0} - \hat{x}_0\right)\left(x_{k_0} - \hat{x}_0\right)^T\right) \quad (20)$$

It is important to notice that the prediction of the state of the system and its associated covariance should include the effect of process noise. The same happens with the prediction of the expected measurement vector and its covariance (innovation covariance), ie, must take into account the effects of observation noise.

The system and measurement noise are assumed to be zero-mean or addictive. This noise implementation does not require the augmenting of the state vector with the noise variables, thus decreasing the number of points that are required to be propagated through the nonlinear system from $2(L+q)$ to $2L$. Being L the dimension of the state of the system at time step k and q the dimension of the state noise process.

The *a priori* mean and covariance of the state are used to calculate the sigma points, defined as:

$$\chi_{k-1} = \left[\hat{x}_{k-1} \quad \hat{x}_{k-1} + \gamma \sqrt{P_{x_{k-1}}} \quad \hat{x}_{k-1} - \gamma \sqrt{P_{x_{k-1}}} \right] \tag{21}$$

where, $\gamma = \sqrt{L + \lambda}$ and λ is the parameter that provides an extra degree of freedom to smooth the higher order moments of the approximation. It can be also used to reduce the overall prediction errors.

The sigma points are chosen to obtain the true mean and covariance of the state distribution and are propagated through a nonlinear system defined as:

$$\chi_{k|k-1} = f\left(\chi_{k-1}, u_{k-1}\right) \tag{22}$$

From the propagated sigma points is also possible to calculate *a posteriori* mean and covariance.

The *posteriori* mean $\hat{x}_k^-$ and covariance $P_{x_k}^-$ are calculated from the statistics of the propagated sigma points by:

$$\hat{x}_k^- = \sum_{i=0}^{2L} W_i^m \chi_{i,k|k-1} \tag{23}$$

$$P_{x_k}^- = \sum_{i=0}^{2L} W_i^c \left(\chi_{i,k|k-1} - \hat{x}_k^-\right)\left(\chi_{i,k|k-1} - \hat{x}_k^-\right)^T \tag{24}$$

being, W_i^m and W_i^c the weights. They are defined as:

$$W_0^m = \frac{\lambda}{L + \lambda} \tag{25}$$

$$W_0^c = \frac{\lambda}{L + \lambda} + 1 - \alpha^2 + \beta \tag{26}$$

$$W_i^m = W_i^c = \frac{1}{2(L + \lambda)} \tag{27}$$

It is possible using the nonlinear measurements model to transform the sigma points, to calculate the estimated measurement matrix $\Upsilon_{k|k-1}$:

$$\Upsilon_{k|k-1} = h\left(\chi_{k|k-1}\right) \tag{28}$$

The mean measurement $\hat{y}_k^-$ and the measurement covariance $P_{y_k y_k}$ are obtained based on the statistic of the transformed sigma points:

$$\hat{y}_k^- = \sum_{i=0}^{2L} W_i^m \Upsilon_{i,k|k-1} \tag{29}$$

$$P_{y_k y_k} = \sum_{i=0}^{2L} W_i^c \left(\Upsilon_{i,k|k-1} - \hat{y}_k \right) \left(\Upsilon_{i,k|k-1} - \hat{y}_k \right)^T + R_{x_k} \tag{30}$$

The cross-correlation covariance $P_{x_k y_k}$ is defined as:

$$P_{x_k y_k} = \sum_{i=0}^{2L} W_i^c \left(\chi_{i,k|k-1} - \hat{x}_k^- \right) \left(\Upsilon_{i,k|k-1} - \hat{y}_k^- \right)^T \tag{31}$$

having the cross-correlation and measurement covariance, the Kalman gain matrix is easily obtained:

$$K_{x_k} = P_{x_k y_k} P_{y_k y_k}^{-1} \tag{32}$$

The update equations are given by:

$$\hat{x}_k = \hat{x}_k^- + K_k \left(y_k - \hat{y}_k^- \right) \tag{33}$$

$$P_{x_k} = P_{x_k}^- - K_{x_k} P_{y_k y_k} K_{x_k}^- \tag{34}$$

where, $\hat{x}_k$ and P_{x_k} are respectively the mean and covariance of the filtered state.

In the unscented method, the filter is initialized with the predicted mean and covariance:

$$\hat{w}_{k_0} = E\left(\hat{w}_0 \right) \tag{35}$$

$$P_{w_0} = E\left(\left(w_{k_0} - \hat{w}_0 \right) \left(w_{k_0} - \hat{w}_0 \right)^T \right) \tag{36}$$

To obtain the time update and covariance of the parameter vector, is necessary to use the following equations:

$$\hat{w}_k^- = \hat{w}_{k-1} \tag{37}$$

$$P_{w_k}^- = P_{w_{k-1}} + Q_{w_k} \tag{38}$$

being, Q_w the system process noise of the time update.

The sigma points are calculated from *a priori* mean and covariance of the parameter through the equations:

$$\Pi_{k-1} = \left[\hat{w}_{k-1} \quad \hat{w}_{k-1} + \gamma \sqrt{P_{w_{k-1}}} \quad \hat{w}_{k-1} - \gamma \sqrt{P_{w_{k-1}}} \right] \tag{39}$$

where, $\gamma = \sqrt{L + \lambda}$ as in the state filter.

The expected measurement matrix, ψ is determined using the nonlinear model:

$$\psi_{k|k-1} = g\left(\Pi_{k-1}, u_{k-1} \right) \tag{40}$$

having the statistics of the expected measurements it is possible to calculate the mean measurement $\hat{d}_k$ and the measurement covariance $P_{d_k d_k}$:

$$\hat{d}_k = \sum_{i=0}^{2L} W_i^{m} \psi_{i,k|k-1} \tag{41}$$

$$P_{d_k d_k} = \sum_{i=0}^{2L} W_i^{c} \left(\psi_{i,k|k-1} - \hat{d}_k\right)\left(\psi_{i,k|k-1} - \hat{d}_k\right)^T + R_{w_k} \tag{42}$$

The cross-correlation covariance, $P_{w_k d_k}$ is obtained using:

$$P_{w_k d_k} = \sum_{i=0}^{2L} W_i^{c} \left(\Pi_{i,k|k-1} - \hat{w}_k^{-}\right)\left(\psi_{i,k|k-1} - \hat{d}_k\right)^T \tag{43}$$

The Kalman gain matrix is approximated from the cross-correlation and measurement covariance using:

$$K_{w_k} = P_{w_k d_k} P_{d_k d_k}^{-1} \tag{44}$$

The measurements update equations are given by:

$$\hat{w}_k = \hat{w}_k^{-} + K_{w_k}\left(d_k - \hat{d}_k\right) \tag{45}$$

$$P_{w_k}^{+} = P_{w_k}^{-} - K_{w_k} P_{d_k d_k} K_{w_k}^{T} \tag{46}$$

i) UKF time update equations:

$$\hat{\mathrm{x}}_k^{-} = \sum_{i=0}^{2L} W_i^{m} \chi_{i,k|k-1}$$

$$P_{\mathrm{x}_k}^{-} = \sum_{i=0}^{2L} W_i^{c} \left(\chi_{i,k|k-1} - \hat{\mathrm{x}}_k^{-}\right)\left(\chi_{i,k|k-1} - \hat{\mathrm{x}}_k^{-}\right)^T$$

ii) UKF measurements update equations:

$$K_{w_k} = P_{w_k d_k} P_{d_k d_k}^{-1}$$

$$\hat{\mathrm{x}}_k = \hat{\mathrm{x}}_k^{-} + K_k\left(\mathrm{y}_k - \hat{\mathrm{y}}_k^{-}\right)$$

$$P_{w_k}^{+} = P_{w_k}^{-} - K_{w_k} P_{d_k d_k} K_{w_k}^{T}$$

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5. Pseudolinearization Based Filtering

This paper proposes a new approach through the pseudolinearization based filtering as an alternative treatment of the data when compared to the classic linearization. This method does not exclude any available data and at the same time avoids the calculus of Jacobian matrices. In this way is possible to have an easier and faster approach.

The conditions for pseudolinearization are obtained in the general case:

$$\dot{x} = A(x)x + B(x)u \tag{47}$$

In a simple way, the following steps can describe this method:

1. Calculate the equations: $\begin{aligned} \dot{x} &= A(x)x + B(x)u + w \\ z &= C(x)x + v \end{aligned}$

2. Calculate the matrices: $\begin{aligned} A_k &= A(\hat{x}_k) \\ B_k &= B(\hat{x}_k) \\ C_k &= C(\hat{x}_k) \end{aligned}$

and apply in the step one: $\begin{aligned} \dot{x} &= A_k x + B_k u + w \\ z &= C_k x + v \end{aligned}$

3. Apply the linear Kalman Filter at one step.

where the main focus is to discover the matrices A(x) and X(x). So, first it is necessary to define the state vector of the system, $x = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6]$ and the control vector u=[u_r u_θ u_ϕ].

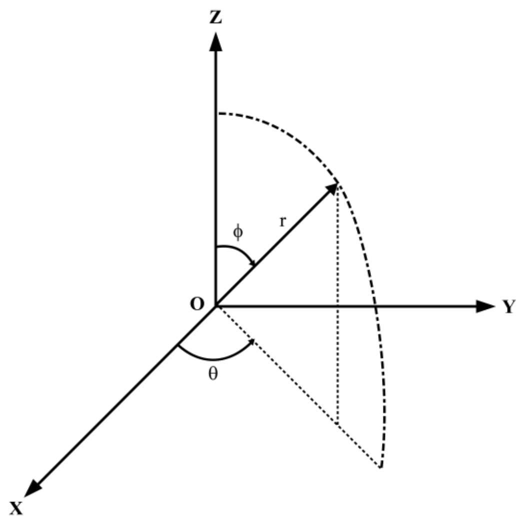

Fig. 1: Motion of satellite in the spherical coordinate system (figure reproduced according with [10]).

The parameters of the state vector, for the case in study and knowing the Earth gravitational field, the thrust accelerations and the inertial spherical coordinates system (figure 1), are:

$$x_1 = r \quad x_2 = \theta \quad x_3 = \phi \quad x_4 = \dot{r} \quad x_5 = \dot{\theta} \quad x_6 = \dot{\phi}$$

being, the equations of motion for a satellite described as:

$$\begin{aligned}
\ddot{r} &= r\dot{\theta}^2 \sin^2\phi + r\dot{\phi}^2 - \frac{\mu}{r^2} + u_r \\
r\ddot{\theta}\sin\phi &= -2\dot{r}\dot{\theta}\sin\phi - 2r\dot{\theta}\dot{\phi}\cos\phi + u_\theta \\
r\ddot{\phi} &= -2\dot{r}\dot{\phi} + r\dot{\theta}^2 \sin\phi\cos\phi + u_\phi
\end{aligned} \tag{48}$$

where, $\mu = GM_e$ is the Earth gravitational constant; M_e is the Earth mass; $G = 6.67\times 10^{-11} Nm^2/kg^2$; u_r, u_θ, u_ϕ are the thrust acceleration components $\hat{i}_r, \hat{i}_\theta, \hat{i}_\phi$ directions respectively; r is the radial distance of the satellite from the center of referential; θ is the angle measured from X-axis in XY plane of an inertial rectangular coordinate system to the projection of r onto XY plane; ϕ is the angle measured from Z-axis to the vector r.

Through equations (48) the matrices A(x) and B(x) are easily defined as (49) and (50).

$$A(x) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ a_{41} & 0 & a_{43} & 0 & a_{45} & a_{46} \\ a_{51} & 0 & a_{53} & a_{54} & a_{55} & a_{56} \\ a_{61} & 0 & a_{63} & a_{64} & a_{65} & a_{66} \end{bmatrix} \tag{49}$$

$$a_{41} = x_5^2 \sin^2 x_3 + x_6^2 - \frac{\mu}{x_1^3}, \quad a_{45} = x_1 x_5 \sin^2 x_3, \quad a_{46} = x_1 x_6, \quad a_{51} = -\frac{2x_4 x_5}{x_1^2}, \quad a_{53} = \frac{2x_5 x_6 \cos x_3}{x_3 \sin x_3}, \quad a_{54} = -\frac{2x_5}{x_1},$$

$$a_{55} = -\frac{2x_4}{x_1} - \frac{2x_6 \cos x_3}{\sin x_3}, a_{56} = -\frac{2x_5 \cos x_3}{\sin x_3}, a_{61} = -\frac{2x_4 x_6}{x_1^2}, a_{63} = \frac{x_5^2 \sin x_3 \cos x_3}{x_3}, a_{64} = -\frac{2x_6}{x_1},$$

$$a_{65} = x_5 \sin x_3 \cos x_3, \quad a_{66} = -\frac{2x_4}{x_1}$$

$$B(x) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & b_{52} & 0 \\ 0 & 0 & b_{63} \end{bmatrix} \tag{50}$$

$$b_{52} = \frac{1}{x_1 \sin x_3}, \quad b_{63} = \frac{1}{x_1}$$

In this way, is possible to execute the pseudolinearization on the available data.

6. Simulation and Results

In this section, we present the simulation results for one example (3D tracking) to compare the different characteristics of the three-nonlinear state estimation method: EKF, UKF and pseudolinearization algorithms.

The simulations of the estimation algorithms were made for one equal system, implemented in Matlab environment. The system is a 3D tracking problem of a Low Earth Orbit (LEO) satellite and the model is implemented in a geocentric reference frame.

The LEO has an inclination of 13º degrees with respect the XY plan and the distance to the perigee and apogee are 6678 km and 7100 km respectively. The LEO was simulated with equations (58) and Butcher algorithm. Its period is 5693 seconds approximately.

It is important to notice that the state vector is given in Cartesian coordinates and the measurement vector is given in spherical coordinates, due to the fact that the sensors are only able to measure the range r, and the angles θ,ϕ.

The process noise and measurement noise applied to the system are Gaussian with zero mean and with a known standard deviation.

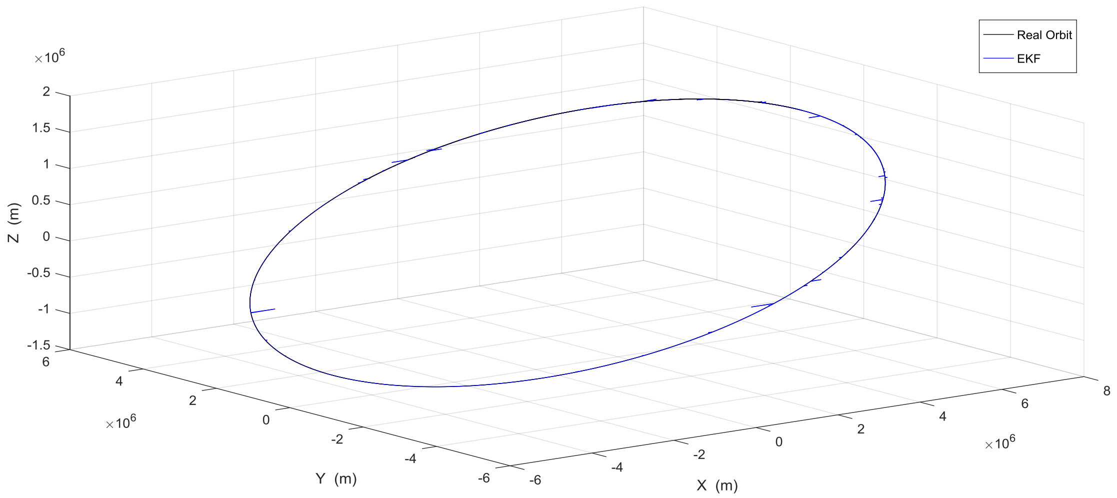

Fig. 2: 3D representation of LEO and the orbit obtained in EKF.

The performance of the EKF is shown in fig. 2. The state vector obtained with EKF is represented in blue and is compared to the real orbit (black trajectory) in a 3D representation (Geocentric frame). In the same figure is possible to observe that the filtered state vector is practically equal to the linear model, which allows to be confident with the filter performance. However, in some points the filter is inconsistent and shows a few peaks.

The poor performance in these points are the direct result of the presence of errors and the poor ability in the algorithm to linearize high nonlinear systems.

Despite that, the EKF is able to predict the sensor readings with very little disturbances, providing a reliable and accuracy data.

Regarding the UKF, the performance is represented in fig. 3. As before, this figure compares the state vector obtained with UKF algorithm (blue trajectory) and the real orbit (black trajectory).

As can be seen, the UKF has a smoother performance, without peaks and presents better results when compared with EKF algorithm.

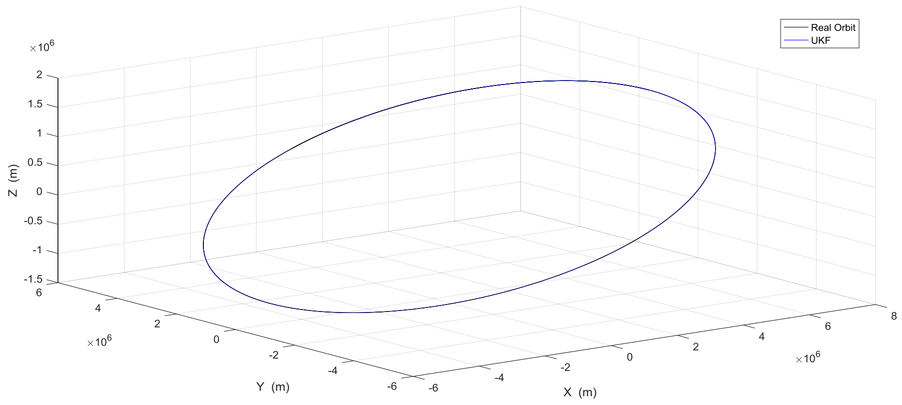

Fig. 3: 3D representation of LEO and the orbit obtained in UKF.

The pseudolinearization performance is represented in fig. 4. As before, the blue line represents the results obtain with the proposed method and the black line the actual orbit.

As can be seen, the pseudolinearization has a better performance than the previous methods, with an improvement of 40% on accuracy.

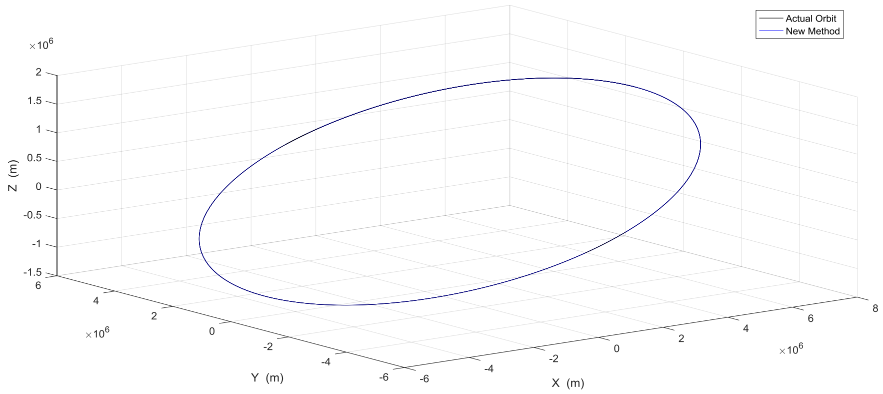

Fig. 4: 3D representation of LEO and the orbit obtained in Pseudolinearization.

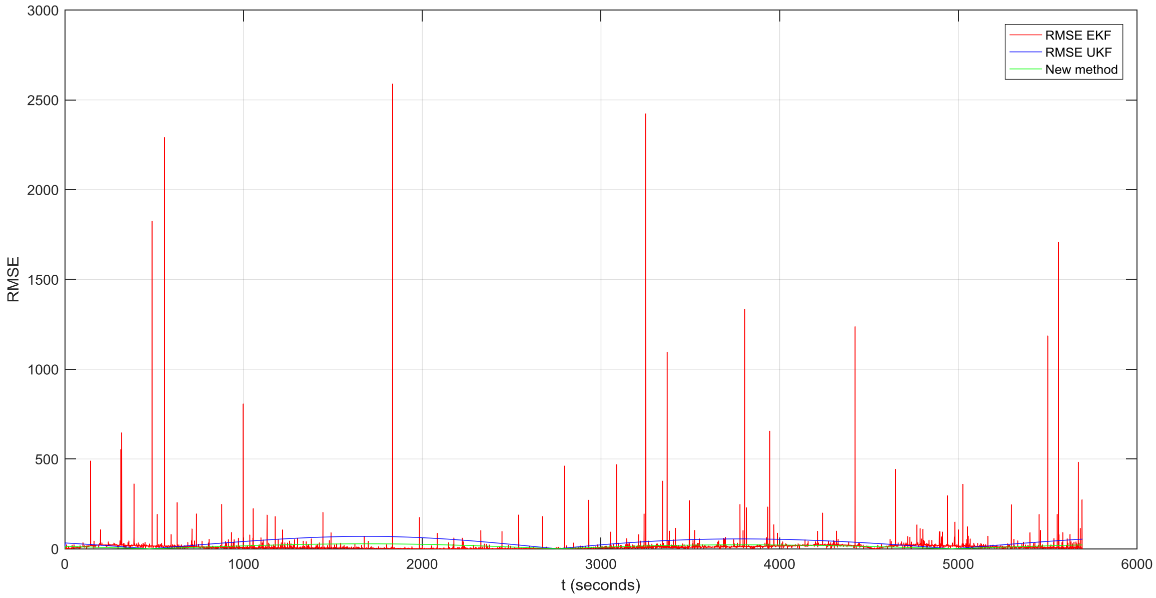

Fig. 5: Root Mean Square Error.

The fig. 5 compare the root mean square error of all methods present in this work. It can be seen that EKF method is inappropriate when the system is highly nonlinear and even though that UKF is better than EKF, this method still presents some issues as being parameter dependent. So, it can be concluded that the method with better performance and accuracy is the pseudolinearization based filtering, with an improvement of 40%.

7. Conclusion

The objective of this paper is to estimate a Low Earth Orbit (LEO) satellite on a radar based orbital trajectory estimation, where the problem to be solved is to smooth noisy data to obtain the best estimate possible.

The system should be able to track the LEO satellite even in an environment that exhibit a set of complicated and highly nonlinear data. Due to non-linear dynamics of the system, the methods used are based in Kalman filter: EKF, UKF and the new method proposed.

It can be concluded that applying just the filter may not be enough to obtain a precise estimate, since the use of just the EKF method proved to be inappropriate when the system is highly nonlinear. The UKF method showed to be better than EKF, however has limitations on being parameter dependent, which sometimes delays all the process of estimation. The pseudolinearization cancels all these problems and permits a more reliable and robust estimate since it is a local linear model that is exactly equal to the original nonlinear one. So, it gives better estimates based on the applications that have been testes so far.

These results support the idea that is attainable to control and maintain the stability of a LEO satellite based on the pseudolinearization.

Still the benefits of this method were demonstrated in a realistic example and has been proved that had an excellent performance in filtering and tracking applications, it is important to notice that this paper has considered one specific form for one set of assumptions, so for future work it is important to continue validating the proposed method on many other nonlinear systems and deal with the mathematics behind the concepts.

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A Brief Author Biography

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