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Effects of Changes of Spokes in the Operation of a Rimmed Flywheel

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Abstract

This study involves the study of effects of changes of spokes in the operation of a rimmed flywheel. The design procedure of a flywheel involves meeting the following constrains/requirements such as fluctuations in speed, fluctuations in energy, space constraints, moment of inertia of flywheel, mass of flywheel, limiting mean rim velocity(centrifugal effect),selection of flywheel material, dimensions of rim and spokes, stresses in flywheel, etc. In this study two types of rimmed flywheel having different cross-sections (Elliptical and circular) were considered and analytical calculations were done as per the design procedure. The effects of changes of spokes in rimmed flywheel was found by varying the number of spokes(2,3,4,5&6).The flywheel was also designed in Solid Works(CAD software) and FEA was performed in FEMAP(FEA software). The credibility of the expected results was established by comparing the CAD and FEA results with analytical results.

Keywords: Rimmed flywheel, Elliptical & circular cross-section, Stresses in flywheel, variation in number of spokes

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1. Introduction

A flywheel is a heavy rotating body that serves as a reservoir of energy. The energy stored in the flywheel is in the form of kinetic energy. The flywheel acts as an energy- bank between the source of power and the driven machinery. Depending upon the source of power and type of driven machinery, there are different applications of the flywheel. Typical applications include punching machines, shearing machines and IC engines.

The functions of a flywheel are:

- To store and release the energy when needed during the work cycle.
- To reduce the power capacity of the electric motor.
- To reduce the amplitude of speed fluctuations.

Basically, there are two types of flywheels. Simple type of flywheel (solid circular disk) and Rimmed flywheel. The solid disk flywheel is rarely used in practice. In most cases, the flywheel consists of a rim, a hub and four to six spokes as shown in Figure 1.

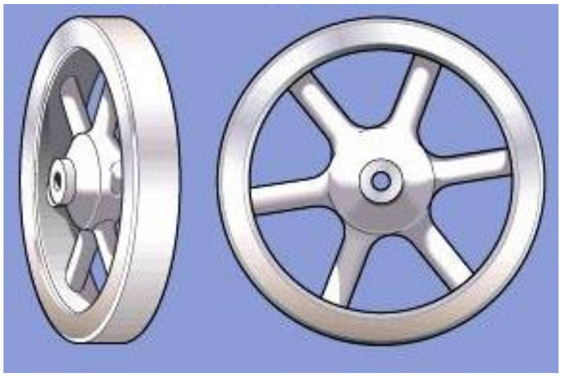


Figure 1: Rimmed Flywheel

2. Problem statement

In order to study the effects of changes of spokes in the operation of a rimmed flywheel, let us consider a problem situation. The flywheel is required to keep down fluctuations in speed from 200 to 220 rpm.The cyclic fluctuations in energy is 30000 N-m, while the maximum torque during the cycle is 75000 N-m. The outside diameter of the flywheel should not exceed 2 m.

Using the above mentioned conditions, we will be designing a rimmed flywheel in such a way that it meets the design standards/requirements.

3. Design Procedure/Analytical calculations

3.1 Determination of required moment of inertia

The average speed of the flywheel is 210 rpm. Therefore,

The mean angular velocity of the flywheel is given by,

$$\omega = \frac{2\pi n}{60} = 21.99\, rad/s \tag{1}$$

The coefficient of fluctuation of speed is given by,

$$C_s = \frac{\omega_{max} - \omega_{min}}{\omega} = \frac{n_{max} - n_{min}}{n} = 0.095 \tag{2}$$

The moment of inertia of the rim is calculated by,

$$I_r = \frac{U_0 K}{\omega^2 C_s} = 587.75\, kg - m^2 \tag{3}$$

Due to the complicated geometric shapes of its component parts, it is difficult to determine the exact moment of inertia of this flywheel. Therefore, the analysis of such a flywheel is done by using anyone of the following two assumptions:

1. The spokes, the hub and the shaft do not contribute any moment of inertia, and the entire moment of inertia is due to the rim alone.
2. The effect of spokes, the hub and the shaft is to contribute 10% of the required moment of inertia, while the rim contributes 90%.

Let us consider K as 0.9. In the design of a flywheel, many times it is required to decide the mean radius of the rim. From equation $I_r = m_r R^2$, it is seen that the mass of the flywheel can be considerably reduced; by increasing the mean radius for a required amount of moment of inertia. The aim should be to use the largest possible radius because it reduces the weight. However, there are two limiting factors- speed and availability of space.

In some cases, the diameter of the flywheel is governed by the amount of space, which is available for the flywheel. Flywheels are usually made of grey cast iron, for which the limiting mean-rim velocity is 30 m/s. When the velocity exceeds this limit, there is a possibility of bursting due to centrifugal force, resulting in an explosion.

Since,

$$v = \omega R \qquad (4)$$

$$R < \frac{30}{\omega}$$

$$R < \frac{30}{21.99} = 1.36\, m$$

Considering the space constraint from the problem statement and limiting mean-rim velocity condition, let us select the mean radius of rim(R) as 0.9m.

The mass of the rim is given by

$$m_r = \frac{I_r}{R^2} = 725.61 \text{ kg} \qquad (5)$$

3.2 Selection of flywheel material

Conventionally, flywheels are made of cast iron. Keeping design considerations in mind, cast iron flywheels has the following advantages.

- Cast iron flywheels are the cheapest.
- Cast iron flywheel can be made into any complex shape without utilizing different machining operations.
- Cast iron flywheel has an excellent ability to damp vibrations.

Table 1 represents the mass density of some of the commonly used flywheel materials.

Table 1 : Mass Density of Flywheel Materials

Material	**Mass Density, kg/m^3**
1. Gray cast iron :	
• FG 150	7050
• FG 200	7100
• FG 220	7150
• FG 260	7200
• FG 300	7250
2. Carbon steels :	7800

Let us consider FG 200 as the flywheel material for further calculations. Table 2 represents the material property of FG 200.

Table 2 : Material property of FG 200

Material	Modulus of elasticity (E)	Poissons's ratio (υ)	Density (ρ)	Tensile strength(σ_{st})
FG 200	114 Gpa	0.26	7100 Kg/m^3	56 Mpa (Considering least value of 0.01% proof stress)

3.3 Determination of dimensions of rim and spokes

The mass of the flywheel rim is also given by,

$$m_r = 2\pi R\left(\frac{b}{1000}\right)\left(\frac{t}{1000}\right)\rho \qquad (6)$$

t=95.06 or 100 mm

b=2t=200mm

The cross-section of the rim is of rectangular type (200 x 100 mm).

Let us consider two types of cross-sections for spokes.

1. Elliptical
2. Circular

In first case it is assumed that the spokes have elliptical cross-section with 200 mm as major axis and 100 mm as minor axis. The cross-sectional area A_1 of the spokes is given by,

$A_1 = \pi ab$

Where, a and b are semi-major and semi-minor axes respectively.

$A_1 = 15707.96\ mm^2$

The cross-sectional area A of the rim is given by,

$A = (200)(100) = 20000\ mm^2$

$$\frac{A}{A_1} = 1.27$$

3.4 Determination of stresses in rim and spokes

A portion of a rimmed flywheel made of grey cast iron is shown in Figure 2.The rotating rim is subjected to a uniformly distributed centrifugal force P_C , which acts in radially outward direction. This induces a tensile force P in the cross-section of the rim acting in tangential direction and a bending moment M. Under the action of centrifugal force, the tendency of the rim is to fly outward, which is prevented due to tensile force P_1 acting in each spoke.

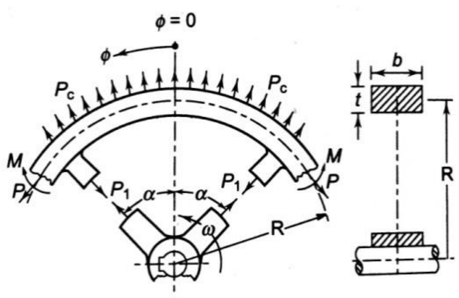


Figure 2

The rim is subjected to direct tensile stress due to P, and bending stresses due to M. The spoke is subjected to tensile stress.

S.Timoshenko’s analysis is based on the following assumptions.

- The thickness of the rim is small in comparison with the mean radius and stresses due to bending moment and tension alone are taken into account; and
- The length of the spoke is considered to be equal to the mean radius, although, in practice, it is somewhat less than the mean radius.

The stresses in the spokes are given by ,

$$\sigma_t = \frac{2}{3}\left[\frac{(1000)mv^2}{CA_1}\right] \tag{7}$$

The stresses in the rim are given by,

$$\sigma_t = \frac{(1000)mv^2}{bt}\left[1 - \frac{cos\emptyset}{3Csin\alpha} \pm \frac{2(1000)R}{Ct}\left(\frac{1}{\alpha} - \frac{cos\emptyset}{sin\alpha}\right)\right] \tag{8}$$

The numerical value of constant C for different number of spokes is calculated by the expression,

$$C = 12\left(10^6\right)\left(\frac{R^2}{t^2}\right) f_2(\alpha) + f_1(\alpha) + \frac{A}{A_1} \tag{9}$$

Where, $f_1(\alpha) = \frac{1}{2sin^2\alpha}\left[\frac{sin2\alpha}{4} + \frac{\alpha}{2}\right]$

$$f_2(\alpha) = \frac{1}{2sin^2\alpha}\left[\frac{sin2\alpha}{4} + \frac{\alpha}{2}\right] - \frac{1}{2\alpha}$$

Where,

m=mass of the rim per mm of circumference (kg/mm)

v=velocity at the mean radius(m/s)

R=Mean radius of rim(m)

t=Thickness of the rim(mm)

b=Width of the rim(mm)

A=Cross-section area of rim(mm^2) (bt)

A_1=Cross-sectional area of spokes(mm^2)

2α=Angle between two consecutive spokes(rad)

C=Constant

The mass m of the rim per millimeter of the circumference is given by,

$$m = bt\rho \tag{10}$$

Where , ρ is the mass density in kg/mm^3

Using above equations, we get

C =3.87

v= ωR=19.79 m/s

The mass of the rim per millimeter of the circumference is given by ,

$$m = bt\rho = \left(\frac{200}{1000}\right)\left(\frac{100}{1000}\right)(7100)(10^{-3}) = 0.142 \text{ kg/mm}$$

Let us consider the calculation for 6 spokes.

(2α)=360/6=60°

(2α) is the angle between consecutive spokes.

Or α=30°=(Π/6) rad

The stresses in the rim @ φ=0° are given by

$\sigma_t = 4.67\ N/mm^2$ (Considering max. value)

The stresses in the rim @ φ=30° are given by

$\sigma_t = 3.47\ N/mm^2$ (Considering max. value)

The stresses in the spokes is given by ,

$$\sigma_t = 0.61\ N/mm^2$$

3.4.1 Tabulation of analytical calculation results

On similar basis, the stresses in rim and spokes are calculated for elliptical and circular cross-sections for different number of spokes. The results are tabulated in tables 3, 4, 5 and 6.

Table 3 : Stresses in rim for elliptical c/s

No. of spokes	Max stress in rim(Mpa)	Angle
2	3.013 3.211	0 90
3	3.297 3.844	0 60
4	3.514 4.409	0 45
5	3.569 4.692	0 36
6	3.468 4.667	0 30

Table 4 : Stresses in spokes for elliptical c/s

No. of spokes	Stress in spokes (Mpa)
2	0.0319
3	0.1368
4	0.3016
5	0.4754
6	0.6108

Table 5 : Stresses in rim for circular c/s

No. of spokes	Max stress in rim(Mpa)	Angle
2	3.009 3.203	0 90
3	3.262 3.771	0 60
4	3.411 4.181	0 45
5	3.408 4.302	0 36
6	3.297 4.199	0 30

Table 6 : Stresses in spokes for circular c/s

No. of spokes	Stress in spokes(Mpa)
2	0.0627
3	0.2547
4	0.5188
5	0.7567
6	0.9188

The graphical representations of the results are as shown in Figure 3 and Figure 4.

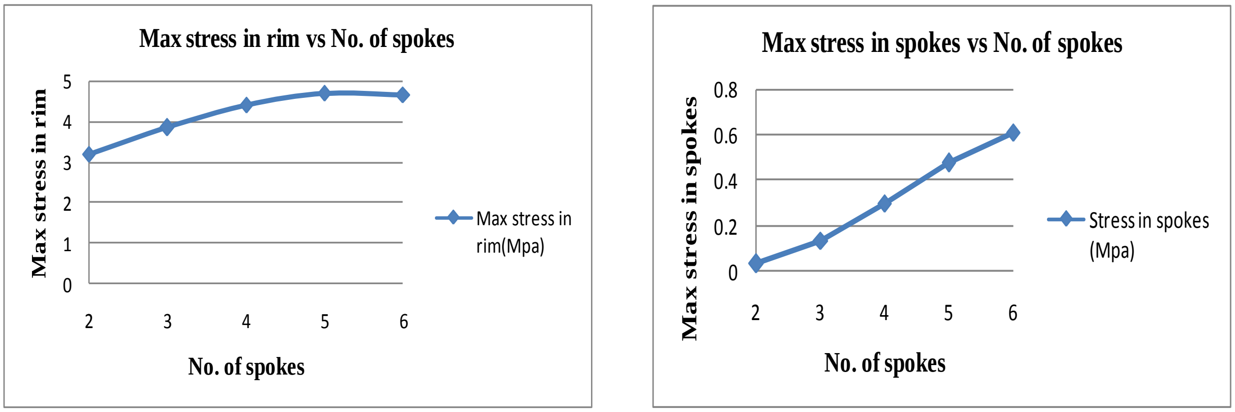


Figure 3: Variation of stress VS number of spokes for elliptical cross-section

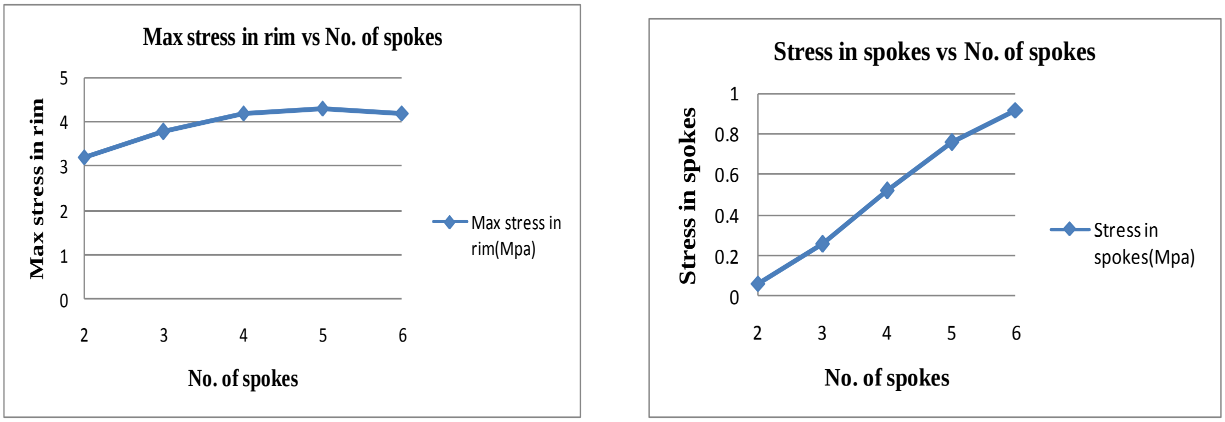


Figure 4: Variation of stress VS number of spokes for circular cross-section

4. Computer Aided Design and Finite Element Analysis

4.1 Computer aided design

The flywheel is designed using Solid works (CAD software).Figure 5 and Figure 6 shows the flywheel consisting of 6 spokes having circular and Elliptical cross-sections respectively.

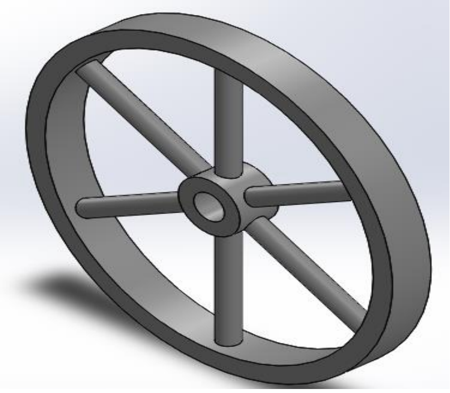


Figure 5: Flywheel having circular cross-section

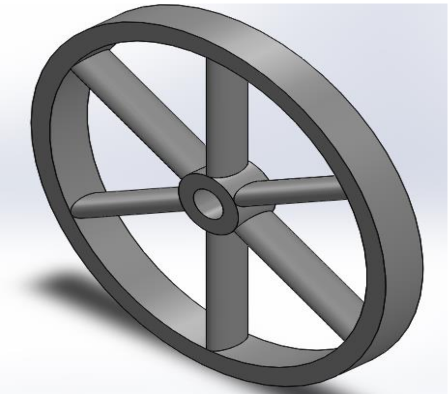


Figure 6: Flywheel having elliptical cross-section

While designing the flywheel in solid works, the hub and shaft design is also considered. Since the shafts are subjected to torsional shear stress, on the basis of strength, plain carbon steel of grade 40C8 is selected. The factor of safety for the shafts is assumed as 2.

The diameter of the shaft is calculated from $T_{max}=\frac{\pi}{16}\tau d^3$ (11)

Where ,

τ= Allowable shear stress for the shaft=95 N/mm^2

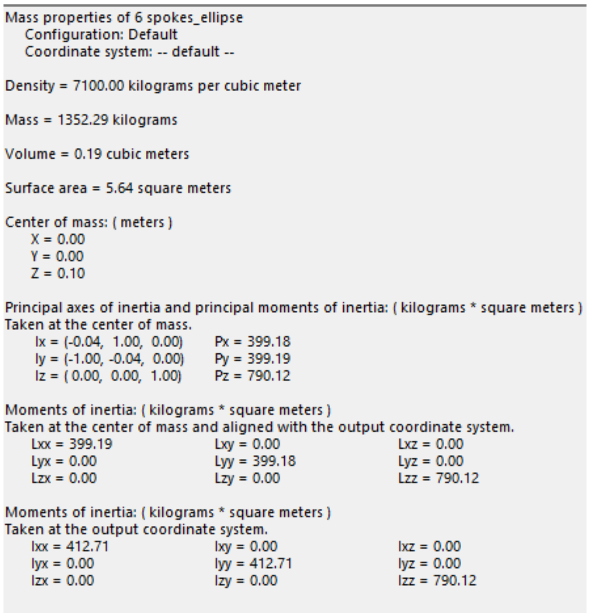
T_{max} = Max. torque during the cycle=75000000 N-mm

Therefore, d= 159.01 mm

Choosing standard shaft diameter as d= 160 mm=0.16 m

Choosing hub diameter as 2x0.16=0.32 m

As snapshot of the mass properties of the 6 spokes elliptical cross-section taken from Solid Works is shown in Figure 7. It is clear that the moment of inertia and mass requirements are met.



```
Mass properties of 6 spokes_ellipse
    Configuration: Default
    Coordinate system: -- default --

Density = 7100.00 kilograms per cubic meter

Mass = 1352.29 kilograms

Volume = 0.19 cubic meters

Surface area = 5.64 square meters

Center of mass: ( meters )
     X = 0.00
     Y = 0.00
     Z = 0.10

Principal axes of inertia and principal moments of inertia: ( kilograms * square meters )
Taken at the center of mass.
      Ix = (-0.04,  1.00,  0.00)        Px = 399.18
      Iy = (-1.00, -0.04,  0.00)        Py = 399.19
      Iz = ( 0.00,  0.00,  1.00)        Pz = 790.12

Moments of inertia: ( kilograms * square meters )
Taken at the center of mass and aligned with the output coordinate system.
     Lxx = 399.19              Lxy = 0.00              Lxz = 0.00
     Lyx = 0.00                Lyy = 399.18            Lyz = 0.00
     Lzx = 0.00                Lzy = 0.00              Lzz = 790.12

Moments of inertia: ( kilograms * square meters )
Taken at the output coordinate system.
     Ixx = 412.71              Ixy = 0.00              Ixz = 0.00
     Iyx = 0.00                Iyy = 412.71            Iyz = 0.00
     Izx = 0.00                Izy = 0.00              Izz = 790.12
```

Figure 7: Mass properties of 6 spokes Elliptical cross-section

4.2 Finite element analysis

The Solid Works computed aided model is imported in FEMAP (FEA software) and further analysis is conducted. The model is meshed using tet elements. The boundary conditions applied are as follows:

- The periphery of the shaft hole is constrained along 5 DOF(Free to rotate about Z-axis)
- A rotational velocity of 21.99 rad/sec about Z axis is applied at co-ordinates (0, 0, 0)
  These boundary conditions simulate the rotation of the flywheel about a particular axis (Z-axis) as shown in figure 8.

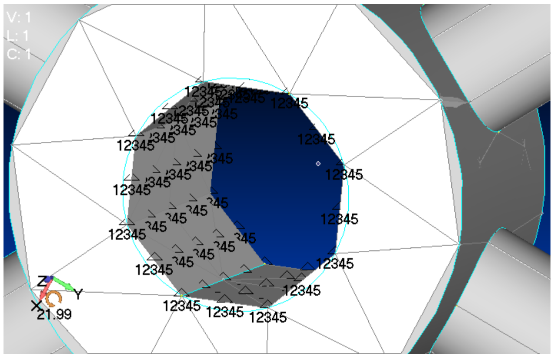


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Figure 8: Boundary conditions

After assigning the material properties, type of elements and boundary conditions, a linear static analysis of the model is performed. The post-processor results of some of the FEA are as shown in Figure 9.

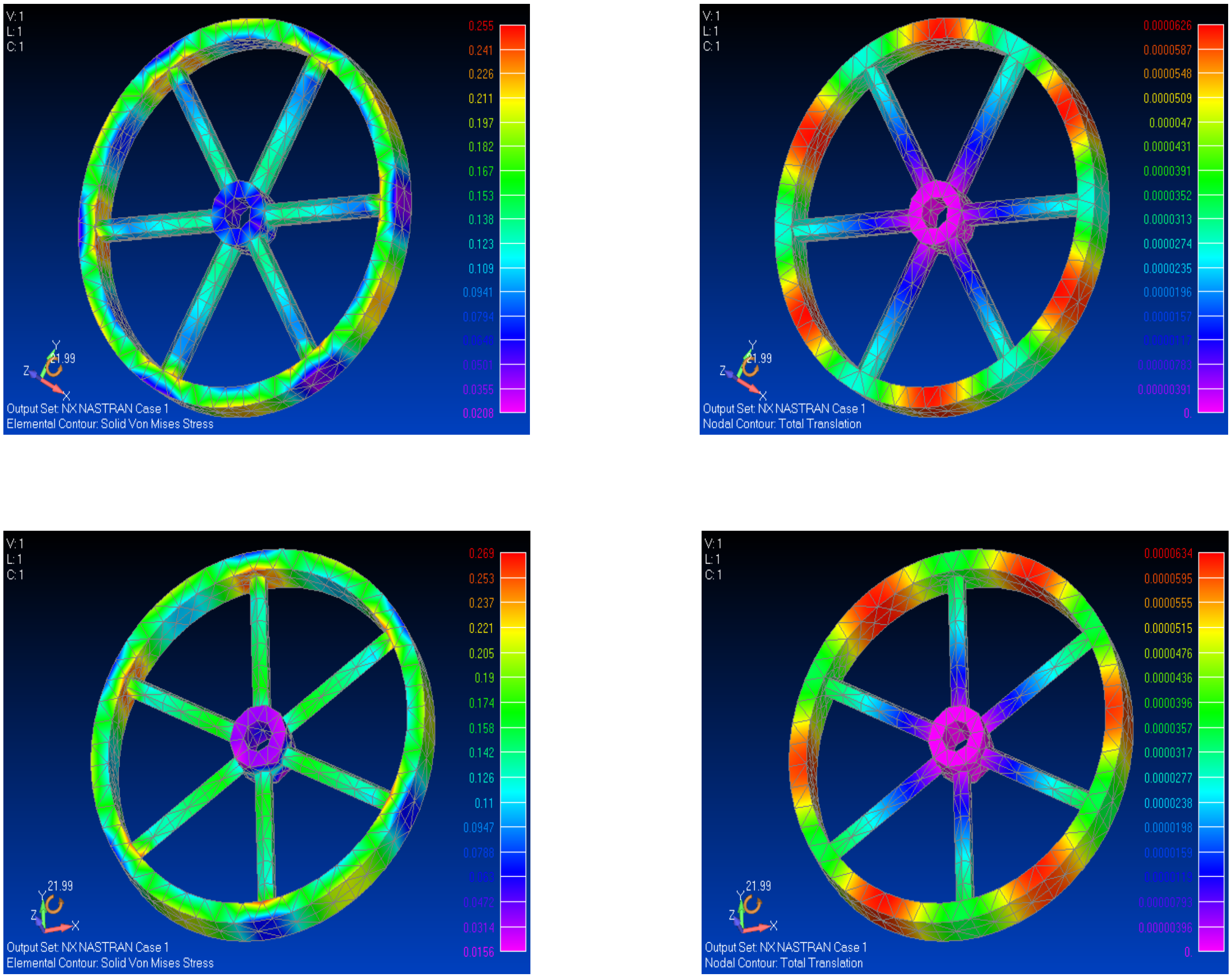

Figure 9: Post-processor results of some of the FEA

4.2.1 Tabulation of FEA results

The stresses and displacements in the flywheel are found from FEA for elliptical and circular cross-sections for different number of spokes. The results are tabulated as shown in tables 7,8,9 and 10.

Table 7 : Stresses in flywheel for elliptical c/s

No. of spokes	Max Stress in flywheel (Mpa)
2	0.232
3	0.266
4	0.27
5	0.266
6	0.241

Table 8 : Displacement in flywheel for elliptical c/s

No. of spokes	Max displacement in flywheel(mm)
2	0.000103
3	0.0000969
4	0.0000857
5	0.0000739
6	0.0000626

Table 9 : Stresses in flywheel for circular c/s

No. of spokes	Max Stress in flywheel(Mpa)
2	0.252
3	0.277
4	0.334
5	0.335
6	0.269

Table 10 : Displacement in flywheel for circular c/s

No. of spokes	Max displacement in flywheel(mm)
2	0.000107
3	0.0000979
4	0.0000833
5	0.0000753
6	0.0000634

The graphical representations of the FEA results are as shown in Figure 10 and Figure 11.

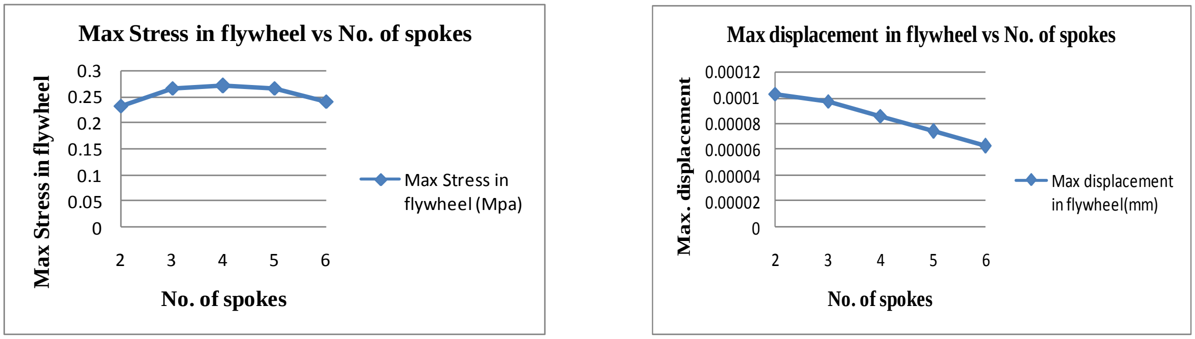

Figure 10: Variation of stress/Displacement VS number of spokes for elliptical cross-section

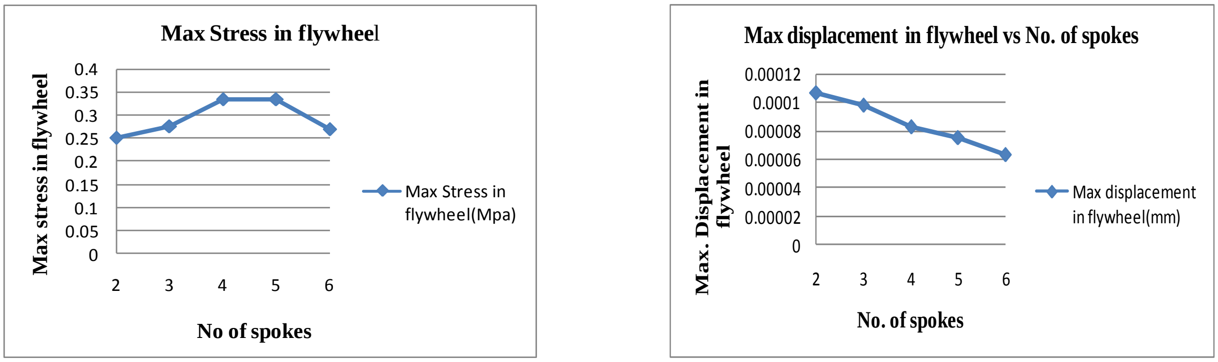

Figure 11: Variation of stress/Displacement VS number of spokes for circular cross-section

5. Conclusion

The design iterations of all the flywheels considered have met the design requirements such as fluctuations in speed, fluctuations in energy, space constraints, moment of inertia of flywheel, mass of flywheel, limiting mean rim velocity(centrifugal effect),selection of flywheel material, dimensions of rim and spokes, stresses in flywheel, etc. which was evident from the analytical calculations.

As expected, the max stress in rim increases as the number of spokes increases. Also, max stress in spokes increases as the number of spokes increases. The max displacement of the flywheel reduces as the number of spokes increases because the flywheel tends to become stiffer. The CAD and FEA results were similar or closer to the analytical results and hence the credibility of the results could be established.

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