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Convergence Study for Material Property Gradient Based Meshing on Analysis of Arbitrary material property variation, Part IV: Under Axial and Transverse Load

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Abstract

In this study a one dimensional functionally graded material (FGM) with arbitrary material property variation that composed of two materials is subjected to axial and transverse load is investigated. Present work considers the elastic property gradient as criteria for meshing and studies its impact on convergence of analysis results. It was observed that elastic gradient basis for element size will have positive effect on convergence if a different gradient relationship is chosen for the meshing. The relationship between the elastic property variation function and the mesh size variation function for an optimum convergence results, Comparative study between transverse and axial loads was carried out for arbitrary material property variation as well as this paper is extension of Convergence study for material property gradient based meshing on analysis of FGMs Part 1 & 2: Under axial loading and transverse loading [1, 2] under IJRAME journal.
Keywords: Graded mesh, Graded element, FGMs, FEM.

1. Introduction

Convergence study for material property gradient based meshing on analysis of functionally graded material Part 1 & 2: Under axial Loading and transverse loading [1, 2] elaborate the $\mathbf{m_{opt}}$ based meshing strategy for the case where the modulus of elasticity E increases or decreases monotonically. Present paper discusses the cases where the material property variation may increase and or decrease along the distance as shown in Figure 1.

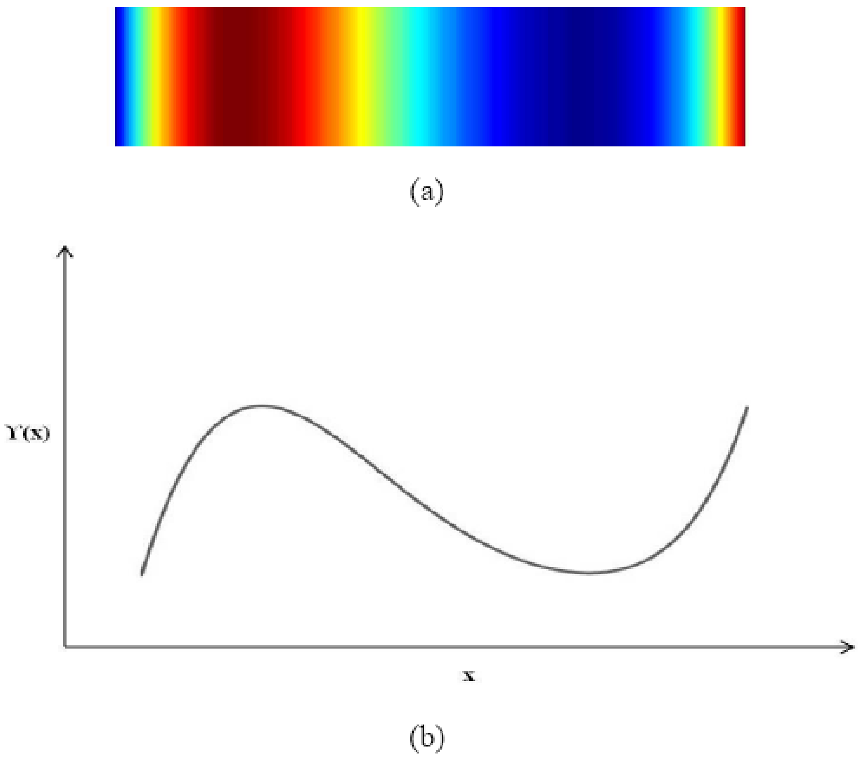

Figure 1: Arbitrary variation FG Material (a) 2D representation of material variation over bar, (b) Material variation shown in Y axis over the length of bar in X axis.

The geometry and loading patterns considered in this paper are the same as were in the case of part 1 and part 2 respectively [1, 2]. The only difference being that the material property variation is different.

The objective of the paper is to see if the $\mathbf{m_{opt}}$ based meshing strategy works satisfactorily for 1 Dimensional bar and beam case.

2. One dimensional bar with $\Upsilon(\mathbf{x})$ Young's modulus variation.

In this paper the problem and assumption as same as part 1 [1], A one dimensional bar problem i.e. length of bar be L, and the area of cross section A as shown in Figure 2 (a), one end of the bar is fixed and a axial load of P is applied at another end. The bar is composed materials which had arbitrary Young's modulus variations over the bar and the variation is govern by Eq. 1, Eq. 2 also Young's modulus variation is shown in Figure 2 (b), (c).

$$\Upsilon(\mathrm{x}) = -4\mathrm{x}^2 + 4\mathrm{x} + 1 \quad (1)$$

$$\Upsilon(\mathrm{X}) = -8.6x^4 + 32x^3 - 33x^2 + 10x + 1 \quad (2)$$

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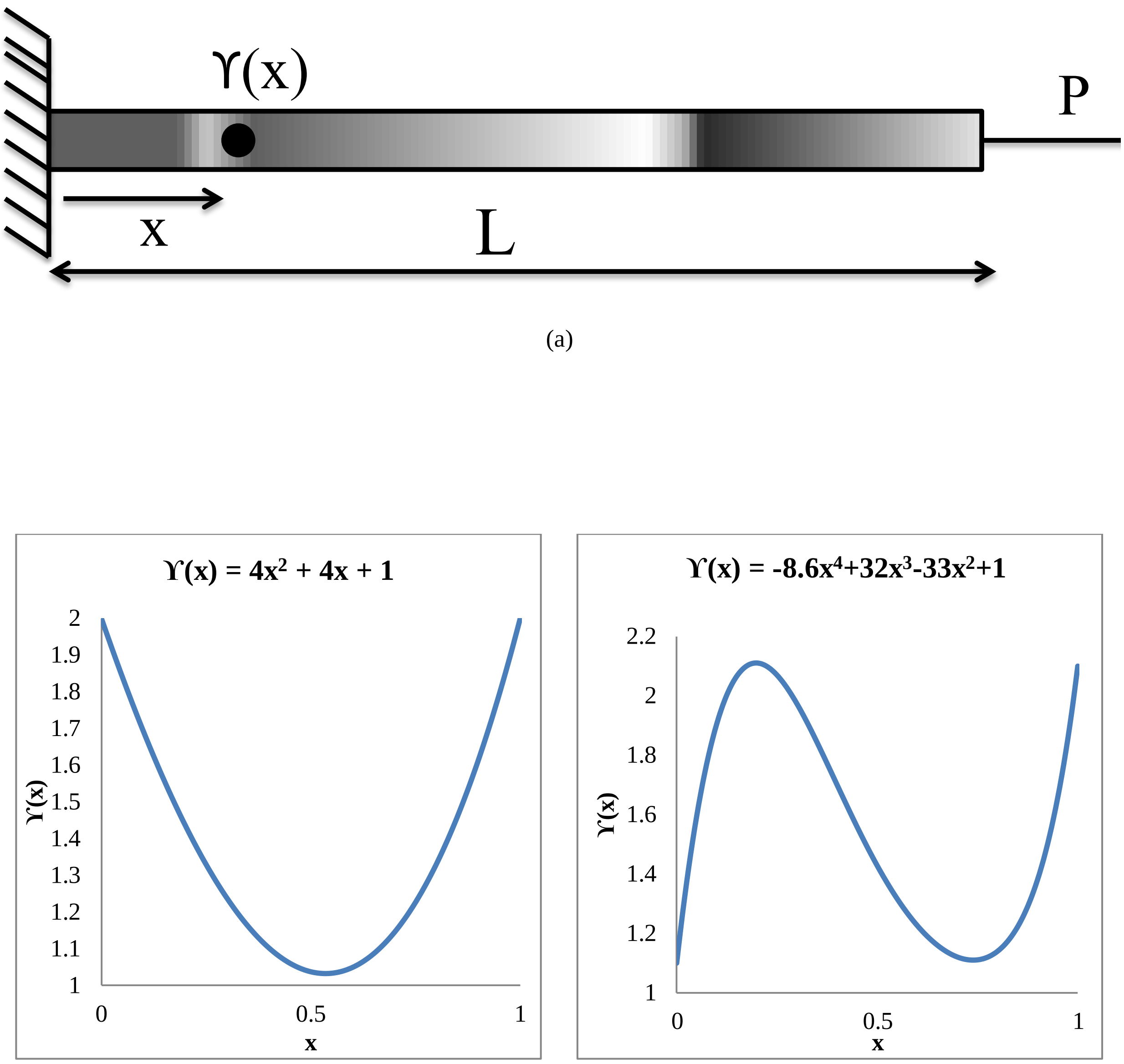

(a)

(b) (c)

Figure 2: Arbitrary variation FGMs bar (a) the basic configuration (b) case 1 (c) case 2

2.1 Closed form solutions for $\Upsilon(x)$ Young's modulus variation

For validation purpose closed form solution of the problem can be taken from standard literature (Timoshenko 1948) and solution method is also same as part 1[1] (i.e. closed form of solution of the problem) only the difference is, in place of Young's modulus variation taken in part 1 [1] (i.e. $\Upsilon(x) = 1 + (\Upsilon_{max} - 1)x^n$) was replace by Young's modulus considers in this paper i.e. Eq. 3 & 4 for making FGM arbitrary variation. And the final equation becomes

$$\frac{du}{dx} = \frac{PL}{A(-4x^2 + 4x + 1)} \quad 3$$

$$\frac{du}{dx} = \frac{PL}{A(-8.6x^4 + 32x^3 - 33x^2 + 10x + 1)} \quad 4$$

2.2 FE analysis with graded Element for $\Upsilon(x)$ Young's modulus variation

As convergence study perform for average element vs. graded element in both the case (graded mesh and geometric mesh) in part 1[1] (comparison of result for average element with graded element) for elemental property for FE analysis and it is concluded that graded element shows good agreement with convergence results, So in this paper Graded element is taken for FE analysis.

The equation of stiffness matrix of an element is

$$[k]_e = \int_{x_1}^{x_2} [B]^T [D][B] A dx$$

Where B is strain displacement matrix and D is material matrix

$$B = \frac{1}{(x_2 - x_1)} [-1 \quad 1]$$

$$D = ⍰(x)$$

$$[k]_e = \int_{x_1}^{x_2} [B]^T [⍰(x)][B] A dx$$

As seen in paper part 3 [3] (mesh generation) where an arbitrary curve is split into number of curves. Generally number of curves are of two type with range of (1 toΥ_{max_L}) & (Υ_{max_L} to 1) with different powers (*n*)

The final element matrix is as follows

If Υ_{max_L} is the maximum value of Υ and occurs at $x_L = 1$.then $\Upsilon(x) = 1 + (\Upsilon_{max_L} - 1)x_L^n$

Else if Υ_{max_L} occurs at $x_L = 1$ then the $\Upsilon(x) = \Upsilon_{max_L} - \left(\Upsilon_{max_L} - 1\right)x_L^n$

$$D = ⍰(x)$$

$$[k]_e = \int_{x_1}^{x_2} \frac{⍰(x)A}{(x_2 - x_1)^2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} dx \quad 5$$

5.3.2 Results and Discussion

Table 1: Displacement results for □(x) $= -4x^2 + 4x + 1$

No of elements	mopt mesh	Graded mesh	Geometrical mesh	Exact Solution
4	0.6145	0.6145	0.6124	
6	0.6175	0.6173	0.6158	
8	0.6185	0.6183	0.6173	0.623225
10	0.6190	0.6188	0.6180	
12	0.6192	0.6190	0.6185	
14	0.6193	0.6192	0.6188	

Table 2: Displacement results for □(X) $= -8.6x^4 + 32x^3 - 33x^2 + 10x + 1$

No of elements	mopt mesh	Graded mesh	Geometrical mesh	Exact Solution
4	0.7466	0.7445	0.7464	
6	0.7501	0.7493	0.7500	
8	0.7513	0.7506	0.7512	0.767828
10	0.7519	0.7514	0.7515	
12	0.7522	0.7519	0.7521	
14	0.7524	0.7521	0.7523	

3. One dimensional beam with $\Upsilon(x)$ Young's modulus variation

In this chapter the problem and assumption is same as part 2 [2], A one dimensional beam problem i.e. length, height, width of beam be L, h, b, and the area of cross section A as shown in Figure 3 (a), one end of the beam is fixed and a transverse load of P is applied at another end. The beam is composed materials which had arbitrary Young's modulus variations over the bar and the polynomial function which govern the Young's modulus variation is expressed by Eq. 6, Eq. 7 and also Young's modulus variation in seen in Figure 3 (b), (c).

□(x) $= -4x^2 + 4x + 1$ 61

□(x) $= -8.6x^4 + 32x^3 - 33x^2 + 10x + 1$ 7

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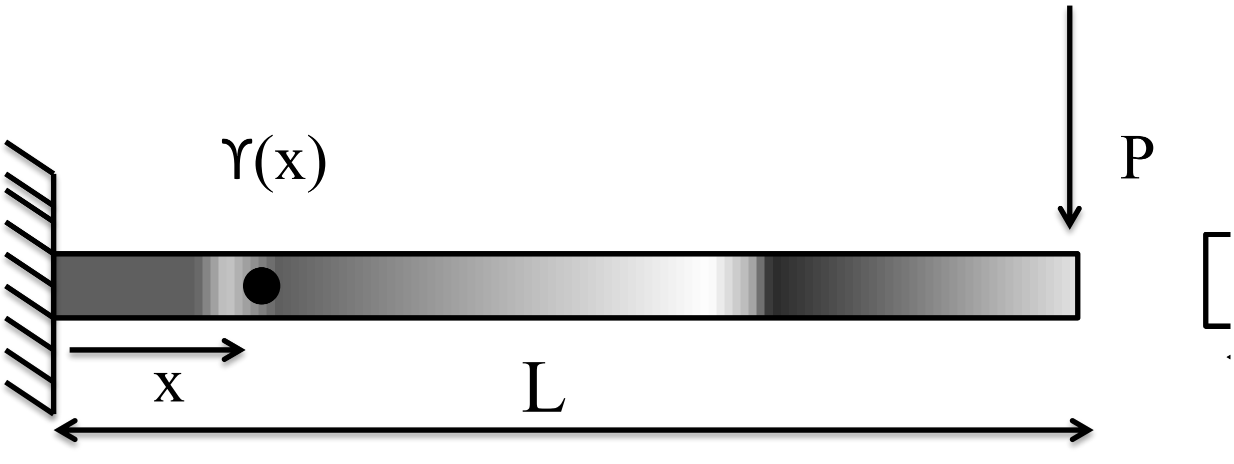
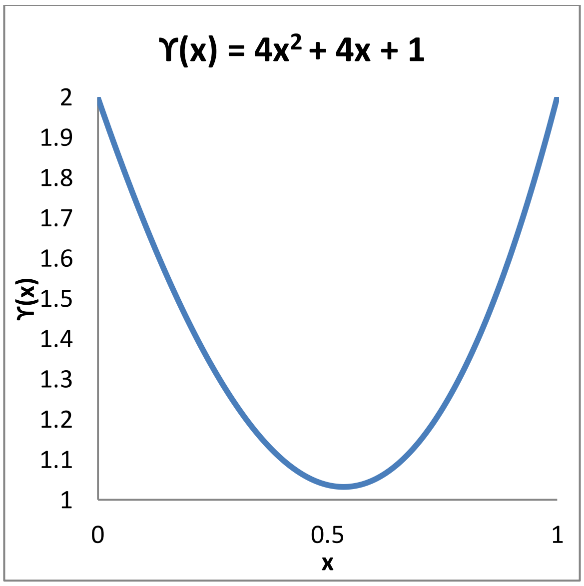


(a)



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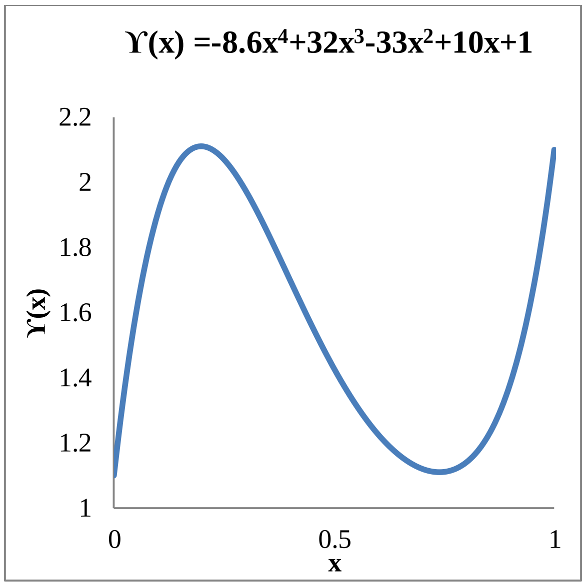

(c)

Figure 3: Arbitrary variation FGMs bar (a) the basic configuration (b) case 1 (c) case 2

3.1 Closed form solutions $\Upsilon(x)$ Young's modulus variation

For validation purpose closed form solution of the problem can be taken from standard literature (Timoshenko 1948) and solution method is also same as part 2 [2] (i.e. Closed form of solution of the problem) only the difference is, in place of Young's modulus variation taken in part 2 [2] (i.e. $\Upsilon(x) = 1 + (\Upsilon_{max} - 1)x^n$) was replace by Young's modulus considered.

And the final equation is as follows.

$$\frac{d^2u}{dx^2} = \frac{P(X) - P(L)}{(-4x^2 + 4x + 1)I} L^2 \quad 8$$

$$\frac{d^2u}{dx^2} = \frac{P(X)-P(L)}{(-8.6x^4+32x^3-33x^2+10x+1)I} L^2 \quad 9$$

3.2 FE analysis with graded Element for $\Upsilon(x)$ Young's modulus variation

As convergence study perform for average element vs. graded element in both the case (graded mesh and geometric mesh) in part 2 [2] (comparison of result for average element with graded element) for elemental property for FE analysis and it is concluded that graded element shows good agreement with convergence results, So in this Section Graded element is taken for FE analysis.

The equation of stiffness matrix of an element is

$$[k]_e = \int_0^l [B]^T [D][B] A dx$$

Whereas B and D is strain displacement matrix and material matrix

As seen in part 3 [3] (mesh generation) where an arbitrary curve is split into number of curves. Generally number of curves are of two type with range of (1 to $\Upsilon_{\max_L}$) & ($\Upsilon_{\max_L}$ to 1) with different powers (n)

The final element matrix is as follows

If Υ_{max_L} is the maximum value of Υ and occurs at $x_L = 1$. then $\Upsilon(x) = 1 + (\Upsilon_{\max_L} - 1)x_L^n$

Else if Υ_{max_L} occurs at $x_L = 1$ then the $\Upsilon(x) = \Upsilon_{\max_L} - (\Upsilon_{\max_L} - 1)x_L^n$

D = □(x)

While substituting the values the final elemental stiffness matrix are as follows

$$[k]_e = I\int_0^l (□(x)) \begin{bmatrix} \left(\frac{12x}{l^3}-\frac{6}{l^2}\right)^2 & \left(\frac{6x}{l^2}-\frac{4}{l}\right)\left(\frac{12x}{l^3}-\frac{6}{l^2}\right) & -\left(\frac{12x}{l^3}-\frac{6}{l^2}\right)^2 & \left(\frac{6x}{l^2}-\frac{2}{l}\right)\left(\frac{12x}{l^3}-\frac{6}{l^2}\right) \\ \left(\frac{6x}{l^2}-\frac{4}{l}\right)\left(\frac{12x}{l^3}-\frac{6}{l^2}\right) & \left(\frac{6x}{l^2}-\frac{4}{l}\right)^2 & -\left(\frac{6x}{l^2}-\frac{4}{l}\right)\left(\frac{12x}{l^3}-\frac{6}{l^2}\right) & \left(\frac{6x}{l^2}-\frac{2}{l}\right)\left(\frac{6x}{l^2}-\frac{4}{l}\right) \\ -\left(\frac{12x}{l^3}-\frac{6}{l^2}\right)^2 & -\left(\frac{6x}{l^2}-\frac{4}{l}\right)\left(\frac{12x}{l^3}-\frac{6}{l^2}\right) & \left(\frac{12x}{l^3}-\frac{6}{l^2}\right)^2 & -\left(\frac{6x}{l^2}-\frac{2}{l}\right)\left(\frac{12x}{l^3}-\frac{6}{l^2}\right) \\ \left(\frac{6x}{l^2}-\frac{2}{l}\right)\left(\frac{12x}{l^3}-\frac{6}{l^2}\right) & \left(\frac{6x}{l^2}-\frac{2}{l}\right)\left(\frac{6x}{l^2}-\frac{4}{l}\right) & -\left(\frac{6x}{l^2}-\frac{2}{l}\right)\left(\frac{12x}{l^3}-\frac{6}{l^2}\right) & \left(\frac{6x}{l^2}-\frac{2}{l}\right)^2 \end{bmatrix} \mathbf{dx} \quad 10$$

3.3 Results and discussion

Table 3: Displacement results for □(x) $= -4x^2 + 4x + 1$

No of elements	mopt mesh	Graded mesh	Geometrical mesh	Exact Solution
4	2.724568	2.72011	2.77104	3.2728
6	2.836878	2.82844	2.82855	

8	2.915401	2.90519	2.91086	
10	2.974905	2.96380	2.97269	
12	3.022196	3.01063	3.02147	
14	3.061038	3.04925	3.06132	

Table 4: Displacement results for □(x) $= -8.6x^4 + 32x^3 - 33x^2 + 10x + 1$

No of elements	mopt mesh	Graded mesh	Geometrical mesh	Exact Solution
4	2.5864	2.6004	2.6032	
6	2.4958	2.5073	2.5142	
8	2.4452	2.4541	2.4622	2.3221
10	2.4122	2.4204	2.4275	
12	23887	2.3957	2.4023	
14	2.3709	2.37702	2.3831	

4 Conclusion

In this paper, checked the $\mathbf{m_{opt}}$ mesh strategy in arbitrary Young's modulus variations results in bar and beam cases and getting same conclusion as part 1 and part 2 respectively. The same $\mathbf{m_{opt}}$ based strategy was applied for two dimensional problems, this time FE software was used.

Reference

[1]. Lokesh Singh (2017); Convergence Study for Material Property Gradient Based Meshing on Analysis of Functionally Graded Materials, Part I - Under Tensile Load, IJRAME
[2]. Lokesh Singh (2017); Convergence study for material property gradient based meshing on analysis of FGMs Part 2: Under Transverse Loading, IJRAME
[3]. Lokesh Singh (2017); Material Property Gradient Based Meshing of Arbitrary material property variation Part 3: An Strategy, IJRAME