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Material Property Gradient Based Meshing of Arbitrary material property variation Part III: an Strategy

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Abstract

In this study a meshing strategy is develop for one dimensional functionally graded material (FGM) with arbitrary material property variation that composed of two or more materials. Present work considers the elastic property gradient as criteria for meshing and studies its impact on convergence of analysis results. It was observed that elastic gradient basis for element size will have positive effect on convergence if a different gradient relationship is chosen for the meshing. The relationship between the elastic property variation function and the mesh size variation function for an optimum convergence results. This paper is extension of Convergence study for material property gradient based meshing on analysis of FGMs Part 1 & 2: Under axial Loading and transverse Loading [1, 2] under IJRAME journal.
Keywords: Graded mesh, Graded element, FGMs, FEM.

1. Introduction

Convergence study for material property gradient based meshing on analysis of functionally graded material Part 1 & 2: Under axial and transverse loading [1, 2] elaborate the $\mathbf{m_{opt}}$ based meshing strategy for the case where the modulus of elasticity E increases or decreases monotonically. Present paper discusses the strategy for handling the cases where the material property variation may increase and or decrease along the distance as shown in Figure 1.

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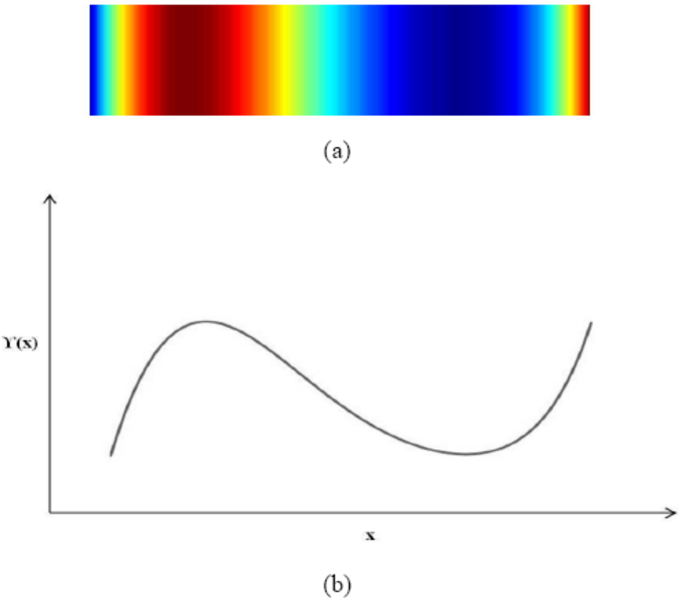


Figure 1: Arbitrary variation FG Material (a) 2D representation of material variation over bar, (b) Material variation shown in Y axis over the length of bar in X axis.

The geometry considered in this paper is the same as were in the case of part 1 and part 2 respectively [1, 2]. The only difference being that the material property variation is different.

2. Problem encountered using the $\mathbf{m_{opt}}$ meshing strategy

As seen in part 1 [1] and part 2 [2], $\mathbf{m_{opt}}$ meshing strategy provided an optimum power $\mathbf{m_{opt}}$ for deciding the graded mesh. The value of $\mathbf{m_{opt}}$ in general is different from the value of the exponent 'n' that defines the material property variation $E(X)$ along the length of the bar/ beam, where

$$E(X) = E_1 + (E_2 - E_1)\left(\frac{X}{L}\right)^n$$

In case the Young's modulus value $E(X)$ is a polynomial expression, for example

$$E(X) = A\left(\frac{X}{L}\right)^p + B\left(\frac{X}{L}\right)^{p-1} + C\left(\frac{X}{L}\right)^{p-2} \dots\dots\dots\dots\dots\dots$$

The above strategy cannot be used directly. So it is obvious that another strategy is required to handle this type of problem.

3. Material and Methods

Divide and Conquer strategy is used to solve this problem. The material distribution curve is split into multiple monotonically increasing or decreasing curves. Each splits curve is approximated as an exponential curve, For each curve depending upon the ratio □ and n, the $\mathbf{m_{opt}}$ is chosen as per the strategy used in part 1 or part 2 [1, 2] and used to mesh that part of the bar/ beam represented by the curve. The strategy is represent for all split curves and each section is separately meshed based on the value of □ as well as the value of n; and thus the value of $\mathbf{m_{opt}}$. The complete strategy is shown in a flowchart in Figure 2.

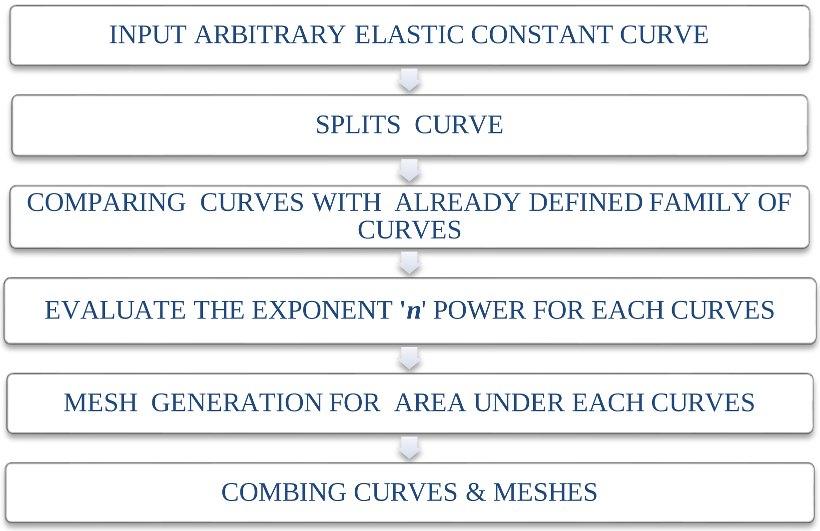


Figure 2: Process Chart of mesh generation for arbitrary FGMs

3.1 Strategy to split the curve

It is presumed that the material distribution can be represented by a continuous polynomial function of the distance.

$$E(x) = A\left(\frac{x}{L}\right)^{p} + B\left(\frac{x}{L}\right)^{p-1} + C\left(\frac{x}{L}\right)^{p-2} + \cdots + R \qquad 1$$

Where A, B, C are constants and depending upon the boundary condition R is E at x=0, i.e. E(0).

In non dimensional form, the same equation can be represented as

$$\Upsilon(x) = ax^{p} + bx^{p-1} + cx^{p-2} + \cdots + 1 \qquad 2$$

Where a, b, c are another set of constants and related to A, B & C

$$as \quad a = \frac{A}{E(0)}, \quad b = \frac{B}{E(0)}, \quad c = \frac{C}{E(0)}$$

So $\Upsilon(x) = ax^{p} + bx^{p-1} + cx^{p-2} + \cdots + 1$ 3

The curve is divided between segments as follows

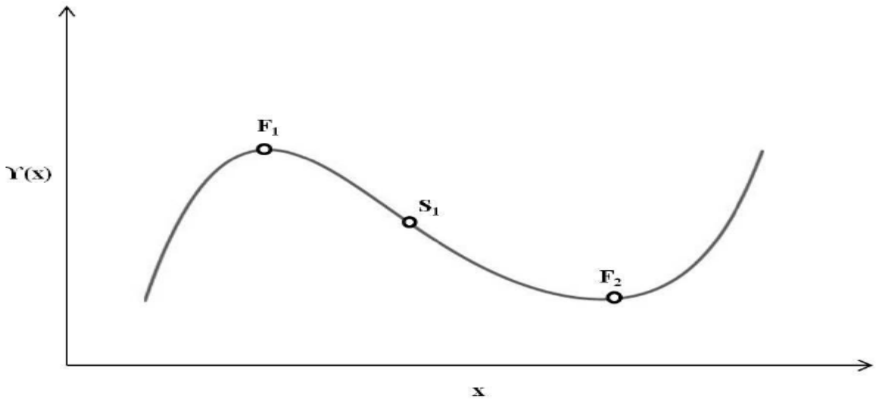


Figure 3: Division of curve based on peak and valley and change in the change slope

Initially the peaks and valley of the curve $\Upsilon(x)$ w.r.t x were captured by equating.

$$\frac{d\Upsilon(x)}{dx} = 0 \qquad 4$$

The points thus obtained are marked as F_1, F_2, F_3………….. as shown in Figure 3.

The curve is further divided on the basis of the change in the sign of the change in slope. For this purpose, Eq. 4 is further differentiated w.r.t. *x* and equating to zero.

$$\frac{d^2\Upsilon(x)}{dx^2} = 0 \qquad 5$$

The points thus obtained are marked as S_1, S_2, S_3…………………….. as shown in Fig 5.3.

The points thus obtained from Eq. 4 and Eq. 5 is arranges in the increasing value of x and marked as x_1, x_2 …..………….. as shown in figure 4. Where x_0 is the initial point & x_L is the last point on the curve representing. The initial point is marked as x_0 and the last point marked as x_L.

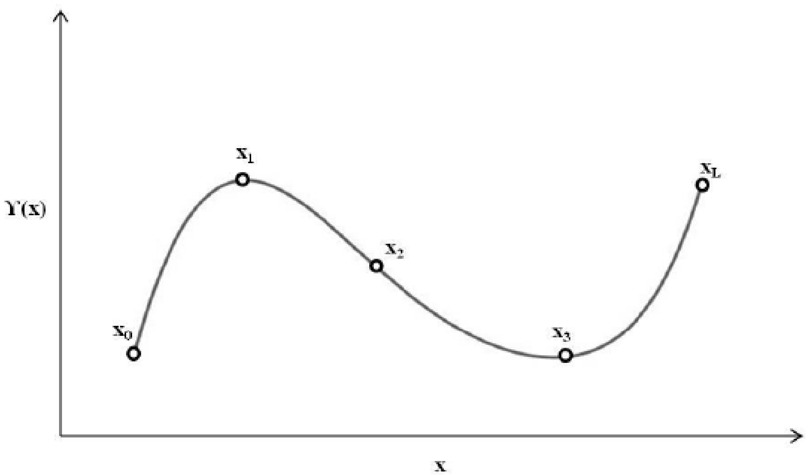
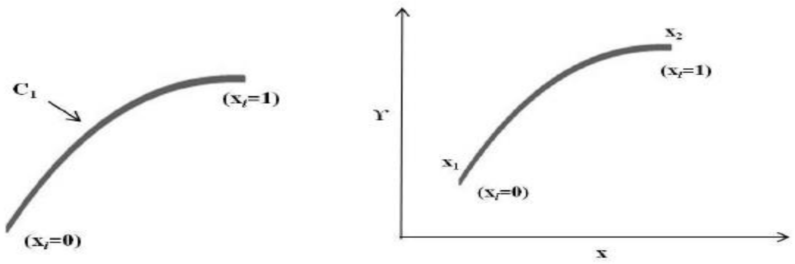


Figure 4: Split curve segments

The curves are thus split in between the incremental values of x thus obtained. For example the curve 1 (C_1) is defined as the part of the curve from $x = 0 \;\; to \;\; x = x_1$ as shown in Figure 5.

The others curve's C_2, C_3 …………etc. are bounded as follows

$(x = 0 \;\; to \;\; x = x_1)$, $\quad (x = x_1 \;\; to \;\; x = x_2) \ldots\ldots\ldots\ldots(x = x_n \;\; to \;\; x = 1)$



(a) (b)

Figure 5: Local definition for the curve segment 1

Each curve segment is now normalized between local values of $x = 0 \;\; to \;\; x = x_1$ and is fitted as shown in Figure 5 in the form of: $\Upsilon(x) = 1 + (\Upsilon_{max_L} - 1)x_L^n$ where Υ_{max} is the maximum value of Υ and occurs at $x_L = 1$.

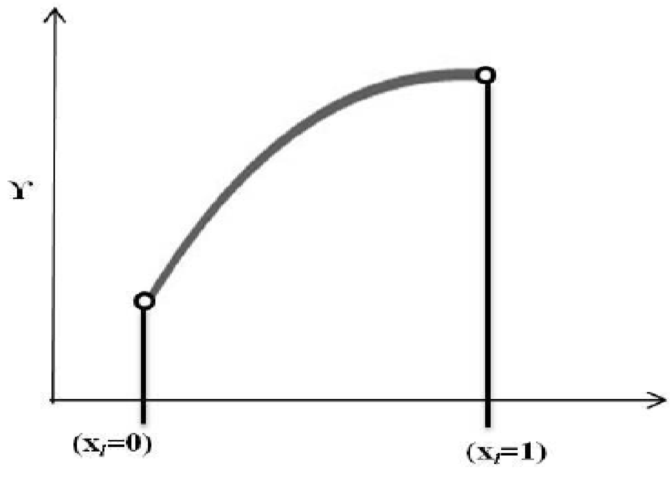


Figure 6: Split curve

For the cases as shown in Figure 6, Υ_{max} occurs at $x_l = 0$ and the equation of $\Upsilon(x)$ becomes as follows $\Upsilon(x) = \Upsilon_{max_l} - (\Upsilon_{max_l} - 1)x_l^n$. The value of '*n'* is evaluated by least square fitting method, taking multiple points along the curve $C_{1.}$

The value of $\mathbf{m_{opt}}$ for the local curve C_1, now can be obtained from the Eq. 6 and 7 respectively for bar or beam as the case may be and is reproducing below

For bar $m_{opt} = \frac{n}{4} + \frac{1}{2}$ 6

And for beam $m_{opt} = \frac{n}{2} + \frac{1}{3}$ 7

The local curve segment C_1 element is now locally meshed based on the local $\mathbf{m_{opt}}$, termed as $\boldsymbol{m_{opt_L}}$. Similarly the entire curve segments are locally meshed and the results of the meshing are combined to get the complete meshing of the object.

3.2 Case Study

For the case, it is presumed that the Young's modulus E variation can be represented by the 4th power of the distance x.

The non dimensional form of polynomial equation

$\Upsilon(x) = -8.6x^4 + 32x^3 - 33x^2 + 10x + 1$, $x \in [\,0 \;\; 1\,]$ 8

ϒ(x) Young's modulus variation is shown in Figure 7.

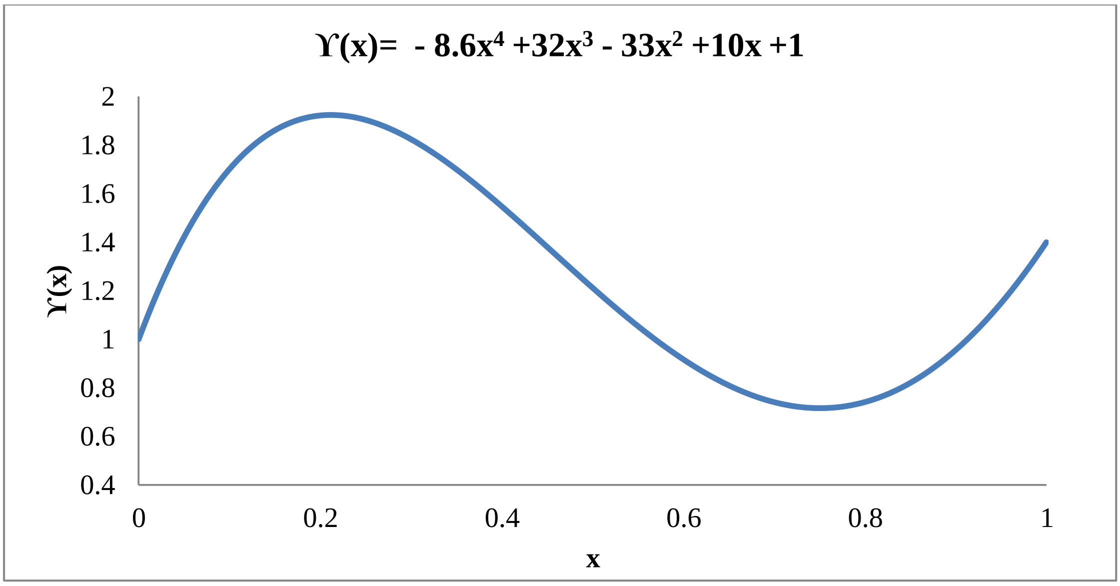


Figure 7: fourth order Young's modulus variation ϒ(x) curve

The curve can be divided between segments as follows:

Initially the peaks and valley of the curve can be obtained by differentiate $\Upsilon(x)$ w.r.t x and equating.

$$\frac{d\Upsilon(x)}{dx} = 0$$ 9

Which yields:

$-34.4x^3 + 96x^2 - 66x + 10 = 0$

After solving, the roots of the polynomial equation are 0.2118, 0.7506 and 1.8282

The point thus obtained are F_1= 0.2118 & F_2=0.7506.

The point 1.8282 is not considered as the range of x is only between 0 and 1

The curve is further divided on the basis of the change in the sign of the change in slope. For this purpose, Eq. 9 is further differentiated w.r.t *x* and equating to zero.

$$\frac{d^2 (x)}{dx^2} = 0 \qquad 10$$

$-103.2x^2 + 192x - 66 = 0$

Which yields:

After solving roots of the polynomial equation is 0.4551 and 1.4054.

The point 1.4054 is again >1 and thus neglected.

The point thus obtained are S_1= 0.4551 (because curve is in range of to 1).

The point thus obtained from Eq. 9 & 10 are arrange in the increasing order value of *x* as shown in Figure 9. Where x_0 is the initial point & x_l is the last point on the curve.

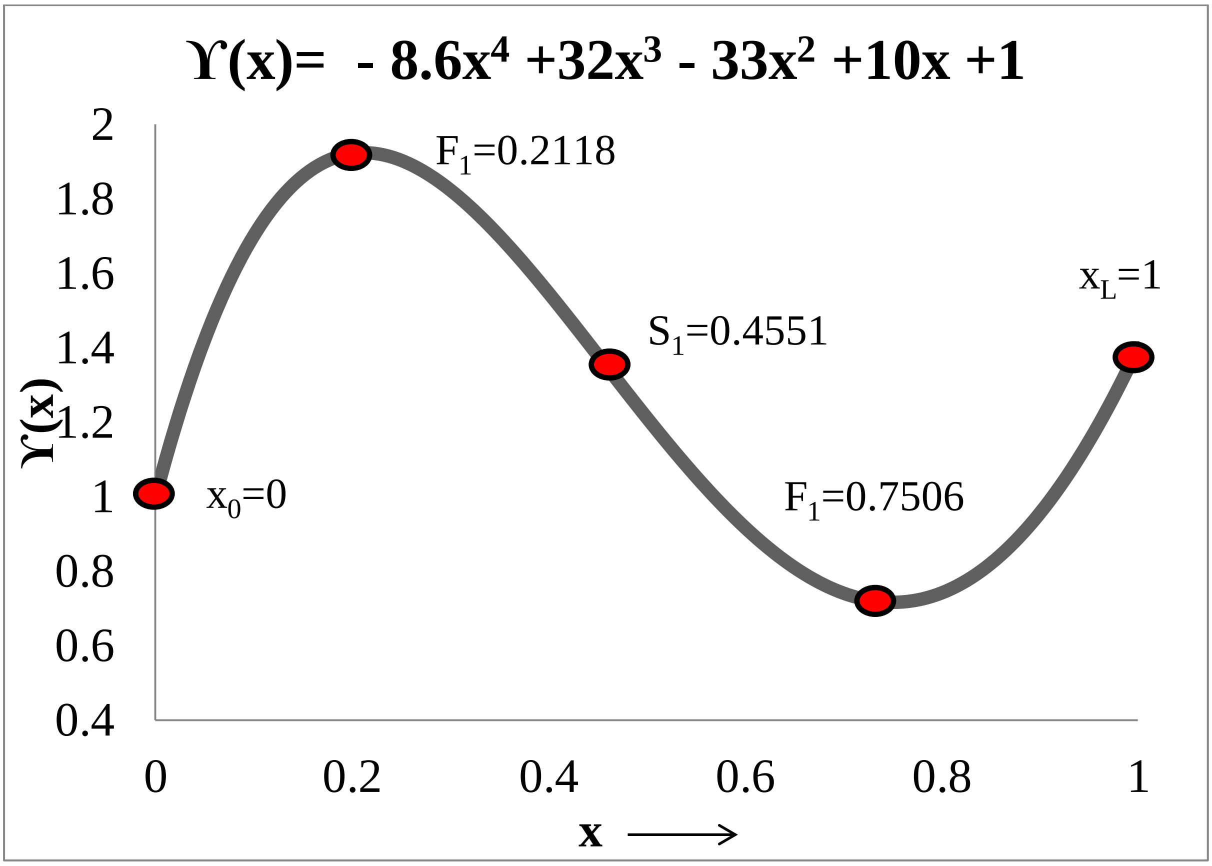


Figure 8: Curve with peak and valley, change in change in slope and initial and end point

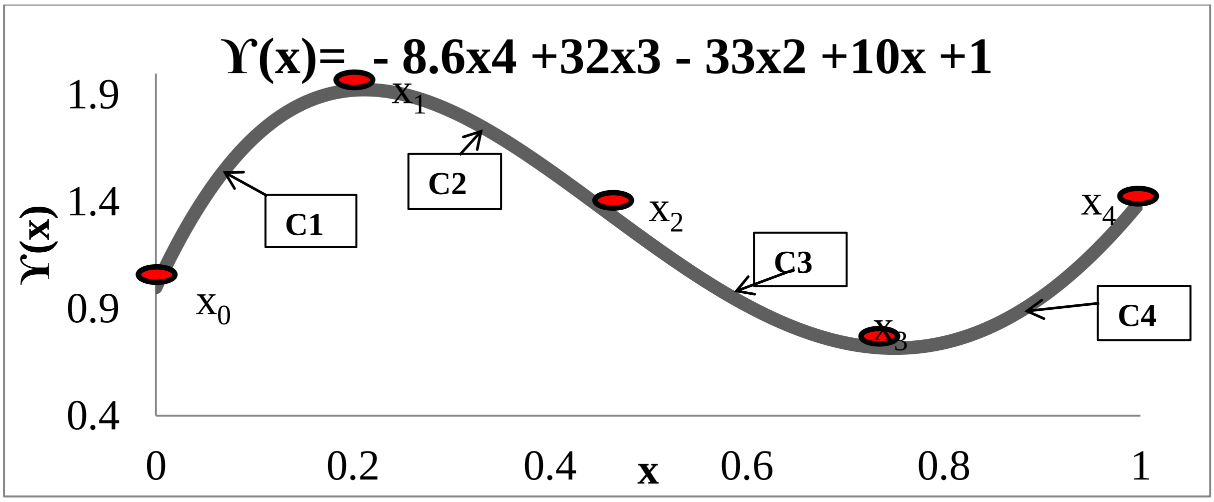


Figure 9: Curve arrange in incremental order

Afterward the curves are split in between the incremental values of *x* as shown in Figure 10 thus obtained as

C_1=x_0 to x_1, C_2=x_1 to x_2, C_3= x_2 to x_3, C_4= x_3 to x_4.

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Afterward mesh generation of each sub region was done using meshing strategy mention in the previous part 1and 2 [1, 2]that are Geometrical mesh, Graded mesh and $\mathbf{m}_{\mathbf{opt}}$ mesh are shown in Table 1 taken as 4 element for each curve.

Using least square fit the change for each section, the value of exponent '*n*' and corresponding '$\mathbf{m}_{\mathbf{opt}}$' is tabulated as follows in table 1

Table 1: Value of exponent and $\mathbf{m}_{\mathbf{opt}}$ of each section

Curve Segment	Range of x	Value of exponent (n)	$\mathbf{m}_{\mathbf{opt}}$ for bar	$\mathbf{m}_{\mathbf{opt}}$ for beam
C_1	$0 \leq x \leq 0.2118$	0.5	0.51	0.56
C_2	$0.2118 \leq x \leq 0.4551$	1.7	1.2	0.85
C_3	$0.4551 \leq x \leq 0.7506$	0.6	0.55	0.59
C_4	$0.7506 \leq x \leq 1$	1.9	1.25	0.92

The plot of the actual material variation and least square fit material variation is shown in Figure 10.

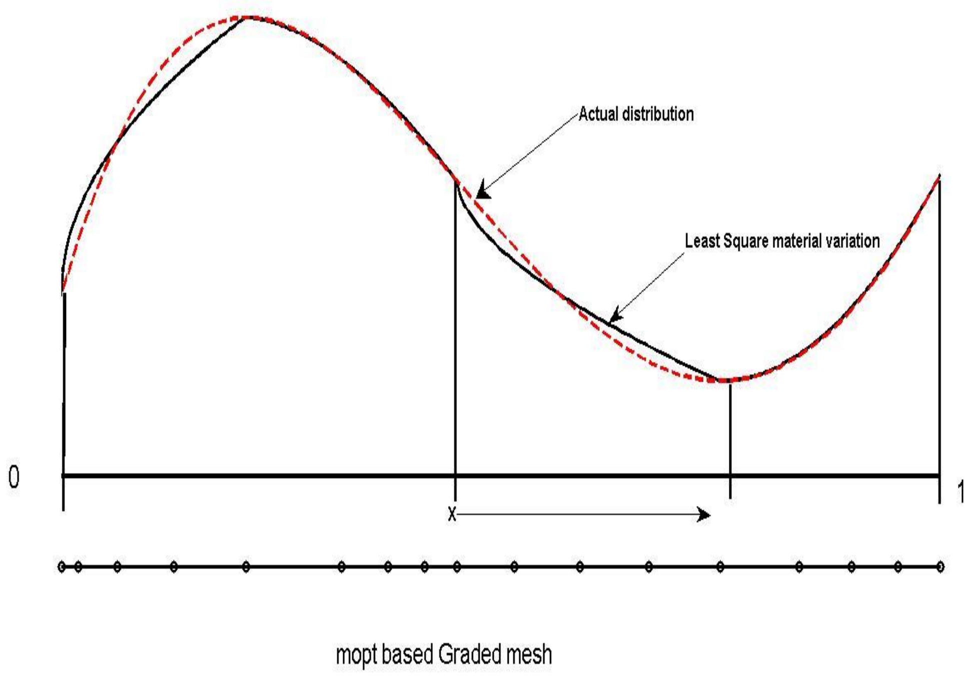

Figure 2: Actual and least square for material variation, $\mathbf{m}_{\mathbf{opt}}$ based graded mesh

5. Conclusion

In this paper, devise a mesh generation method for arbitrary Young's modulus variation of FGMs as well as checked the $\mathbf{m_{opt}}$ mesh generation strategy in arbitrary Young's modulus variations. The same $\mathbf{m_{opt}}$ based strategy was applied for analysis purpose in 1 dimensional bar and beam problems.

6. Reference

[1]. Lokesh Singh (2017); Convergence Study for Material Property Gradient Based Meshing on Analysis of Functionally Graded Materials, Part I - Under Tensile Load, IJRAME

[2]. Lokesh Singh (2017); Convergence study for material property gradient based meshing on analysis of FGMs Part II: Under Transverse Loading, IJRAME