INTERNATIONAL JOURNAL OF RESEARCH IN AERONAUTICAL AND MECHANICAL ENGINEERING
WWW.IJRAME.COM
ISSN (ONLINE): 2321-3051

Vol.5 Issue.5,
May 2017
Pg: -33-45

Convergence study for material property gradient based meshing on analysis of FGMs Part 2: under Transverse Loading

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Abstract

In this study a one dimensional functionally graded material (FGM) object that composed of two materials is subjected to transverse load is investigated. Material property is varied continuously in term of two materials volume fraction. Present work considers the elastic property gradient as criteria for meshing and studies its impact on convergence of analysis results. It was observed that elastic gradient basis for element size will have positive effect on convergence if a different gradient relationship is chosen for the meshing. The relationship between the elastic property variation function and the mesh size variation function for an optimum convergence results, Comparative study between transverse and tensile loads was carried out and generalized an optimal relationship between elastic property variation function and mesh size variation function for one dimensional FGM object.

Keywords: Graded mesh, Graded element, FGMs, FEM.

1. Introduction

FGMs are microscopically in homogeneous composite material, in which elastic properties vary continuously and smoothly from one end to another [1]. Because of additional possibilities FGM offers for optimizing the design of component in terms of material usage and performance, FGM have found many special application for the possibilities of material that can withstand very high thermal gradient like in aerospace, nuclear, automotive, computer circuit industries and for ballistic purpose and percussive as well as biomedical applications [2, 3].

Much literature available on static dynamic analysis for FGM structures in the area wave propagation behavior, thermal behavior, vibration behavior, structural behavior, fracture parameters etc [4]. And also in FE analysis FE element is taken based on available theories. Much work is done in the study of non linear analysis, mechanical behavior, non linear bending response, non linear free vibrations, and FEA of FGMS structures under transverse load [1], [5-8]. But the effect of mesh size while in analysis of FGM object is largely untouched.

Similar to design methods, the computational simulations are important tool of development because of their potential to reduce expensive experimentation. Finite element simulation is a vital tool for analysis and mesh generation is an essential part of it. Extensive work is available in the area of mesh generation algorithms and handling complex geometric objects but only few papers are available on meshing based on varying material property termed here as graded mesh. Cheng et. al [9] has put parent pattern module method to generate the graded mesh, by allowing for gradation in both coordinate directions. They generated quadrilateral elements, with no restriction on the distribution of mesh density. Peter et. al [10] presents an algorithm which can be used to triangulate geometries represented by parametric coordinates. The parameterization used to define all curves as non uniform rational B-Spline (NURBS) to automatically triangulate a region with a graded mesh. Mezzanotte et. al. [11] focused on a graded meshing strategy to minimize the coarseness error. They defined two types of cell sizes, large and small and graded mesh is applied from small size cell that increased with a constant factor to large cell size. Apart from homogeneous object meshing, now a day’s researchers are taking a keen interest in functionally graded material meshing. Chiv et. al [12] implemented quad-tree mesh generation method to separate the interface region of different materials within an object and generated a triangular mesh. Zhang et. al [13] describes an approach for automatic unstructured tetrahedral and hexahedral meshes of composite domain made up of heterogeneous materials by introducing the notation of a material change edge and minimizer point method for identifying the interface boundary and interface node between different materials. Nicole. et.al [14] separated different component of the cervical spine with a triangulated surface region of the structure and using a multi-block method for mesh generation. Sullivan [15] developed a three dimensional mesh generation method which is well suited for adaptive situations. A template of elements is superimposed upon the boundary of the model and elements that are straddled at the boundary are adapted to conform to the models boundaries. Internal and distinct materials are retained in the final mesh.

FE analysis of heterogeneous object is relatively current topic in research. Pise et. al [16] Simulated static loading of bio-objects like human femur with B-Spline based modeling, meshing and its 3D finite element analysis with material based graded element. Pfeiler et. al [17] employed a direct conversion of CT Hounsfield units to material property (young modulus and poisons ratio) and minimize user interaction for mesh smoothing to produce FEM analysis model. Yang et. al [18] proposed heterogeneous lofting for modeling of a multi-material object and for analysis graded B-Spline finite element solution procedure which gives better convergence magnitude.

Most of the works stop at adopting some criteria or the other related meshing the object. No study is available for checking the effect of these criterions on the analysis of the object. Present work uses material gradient as one of the parameter to create variable mesh and studies its effect on the convergence characteristics of a FEM analysis procedure.

2. Materials and Methods

2.1. Problem Statement

We start with a very basic and well known one dimensional object example. Let length of object be L, height of the obect is h and the breadth of the object is b. As shown in Figure 1, one end of the object is fixed and transverse load P is applied at another end. The object is composed of two materials, with the elastic properties assumed to be E_1 and E_2 respectively ($E_2 > E_1$).The variation of modulus elasticity E(X) at a distance X from the fixed end within the beam is govern by a power law function as follows.

$$E(X) = E_1 + (E_2 - E_1)\left(\frac{X}{L}\right)^n \tag{1}$$

Let $x = \frac{X}{L}$, $\gamma_{max} = \frac{E_2}{E_1}$, & $\gamma(x) = \frac{E(x)}{E_1}$

Where x is termed as non dimensional length and $\gamma(x)$ as non dimensional modulus of elasticity using non dimensional parameters Eq. 1 reduces to

$$\gamma(x) = 1 + (\gamma_{max} - 1)x^n \tag{2}$$

Where n is any real positive number called material variation power. For all simulation it is presumed that the height h, breadth b and load P is 1unit.

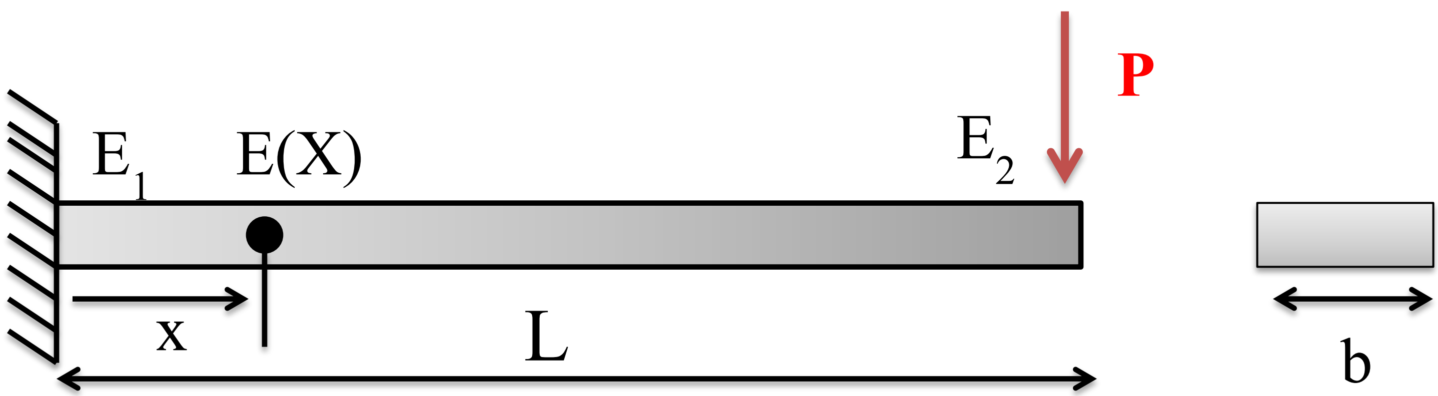


Figure 1: The basic configuration

The material property distribution within the longitudinal direction for different values of n is shown in Figure 2.

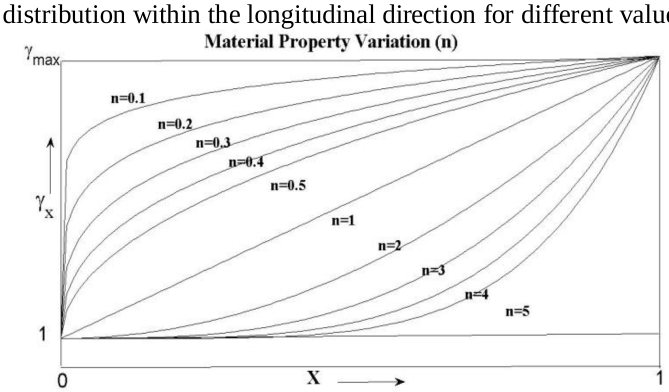


Figure 2: Material property variation

2.2. Meshing style

For homogeneous objects, the mesh size is dependent primarily on the geometry of the object. It is presumed that for FGM, the appropriate mesh size is dependent on the material property gradient. The material property considered here is the modulus of elasticity of the object, i.e. $E(X)$.

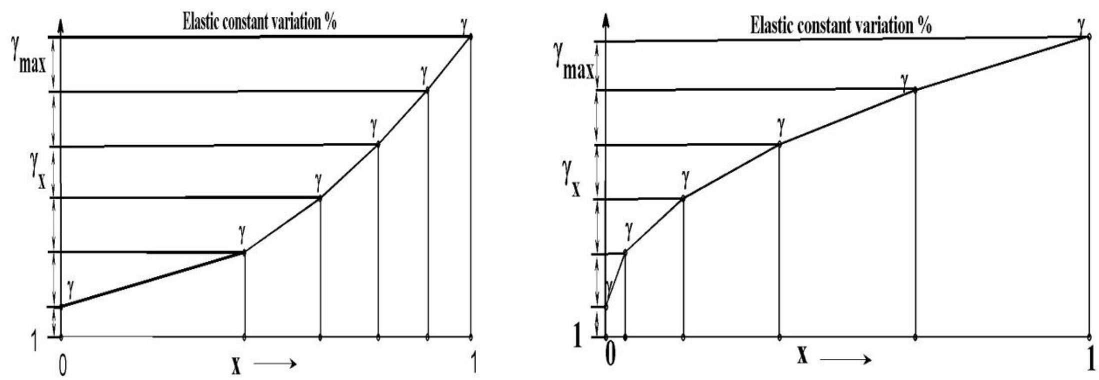
Let the rod be divided into N number of elements for geometric mesh, since the area of cross section is constant, all elements are considered of equal size. Thus the non dimensional length of each element is$1/N$. A standard finite element formulation was done and displacement u at the load end of the beam was checked for convergence study.

The second study was done using the material based graded meshing approach as follows. Let the number of elements be N. Now the element size will be determined by equal increment in modulus of elasticity (E) along X direction. Each node will have an increment of$(E_2 - E_1)/N$. So the nodes will be placed at the locations of successive increment of $(E_2 - E_1)/N$ in the value of E. Let the value of E at a node be $E(X)$.

.The non dimensional distance corresponding to non dimensional modulus of elasticity can be determined by:

$$x = \left(\frac{\gamma(x)-1}{\gamma_{max}-1}\right)^{\frac{1}{n}} \tag{3}$$

The different element length for different values of n is shown in Figure 3(a) - (b).



(a) (b)

Figure 3: Location of nodes based on equal variation in E (a) for n=2 and (b) n=1/2

3. FE Analysis with Graded Element

For an object with variable axial parameter, the parameter is E(X) is varying with axial coordinate X. The axial parameter can have different variation forms, while they can formulate in terms of power function in equation 1. Taking equation 1 the stiffness matrix [k]e of element are written as

$$[k]_e = \int_0^l [B]^T [E(X)][I][B] A dX \tag{4}$$

Where as in present study the variation in elastic property with an element can be handled as method termed here as graded element where the value of E varying across the element as shown in Figure 4 & 5.

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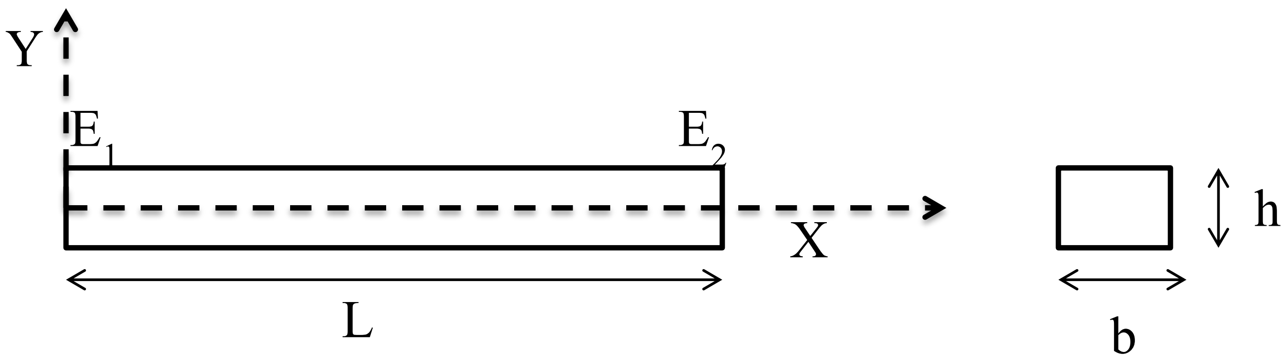

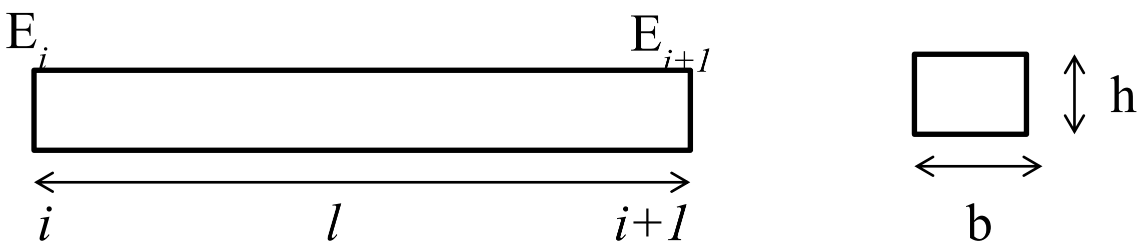

Figure 4: Coordinate system of complete object.

Figure 5: Schematic of element with varying E

Whereas l is the length of element, E is a function of X and elastic property in the element [19] can be written as

$$E = E_i + (E_{i+1} - E_i)\left(\frac{X}{L}\right)^n \qquad (5)$$

Whereas $B = \left[\frac{-6}{l^2} + \frac{12x}{l^3},\ \frac{-4}{l} + \frac{6x}{l^2},\ \frac{6}{l^2} - \frac{12x}{l^3},\ \frac{6x}{l^2} - \frac{2}{l}\right]$ as strain displacement matrix

$$[k]_e = I\int_0^l \boldsymbol{E_i + (E_{i+1} - E_i)x^n} \begin{bmatrix} \left(\frac{12x}{l^3} - \frac{6}{l^2}\right)^2 & \left(\frac{6x}{l^2} - \frac{4}{l}\right)\left(\frac{12x}{l^3} - \frac{6}{l^2}\right) & -\left(\frac{12x}{l^3} - \frac{6}{l^2}\right)^2 & \left(\frac{6x}{l^2} - \frac{2}{l}\right)\left(\frac{12x}{l^3} - \frac{6}{l^2}\right) \\ \left(\frac{6x}{l^2} - \frac{4}{l}\right)\left(\frac{12x}{l^3} - \frac{6}{l^2}\right) & \left(\frac{6x}{l^2} - \frac{4}{l}\right)^2 & -\left(\frac{6x}{l^2} - \frac{4}{l}\right)\left(\frac{12x}{l^3} - \frac{6}{l^2}\right) & \left(\frac{6x}{l^2} - \frac{2}{l}\right)\left(\frac{6x}{l^2} - \frac{4}{l}\right) \\ -\left(\frac{12x}{l^3} - \frac{6}{l^2}\right)^2 & -\left(\frac{6x}{l^2} - \frac{4}{l}\right)\left(\frac{12x}{l^3} - \frac{6}{l^2}\right) & \left(\frac{12x}{l^3} - \frac{6}{l^2}\right)^2 & -\left(\frac{6x}{l^2} - \frac{2}{l}\right)\left(\frac{12x}{l^3} - \frac{6}{l^2}\right) \\ \left(\frac{6x}{l^2} - \frac{2}{l}\right)\left(\frac{12x}{l^3} - \frac{6}{l^2}\right) & \left(\frac{6x}{l^2} - \frac{2}{l}\right)\left(\frac{6x}{l^2} - \frac{4}{l}\right) & -\left(\frac{6x}{l^2} - \frac{2}{l}\right)\left(\frac{12x}{l^3} - \frac{6}{l^2}\right) & \left(\frac{6x}{l^2} - \frac{2}{l}\right)^2 \end{bmatrix} \boldsymbol{Ldx} \qquad (6)$$

Elemental equations are assembled to get global properties of structure by using following system equation

$$\{k\}_e\{\delta\}_e = \{P\}_e \qquad (7)$$

Where [k]e is element stiffness matrix, {δ}e is nodal displacement vector of the element and {P}e nodal load vector. The global equation can be presented as

$$[K][\delta] = [P] \tag{8}$$

Where [K] is the global stiffness matrix [δ] is the global displacement vector and [P] is the global load vector

3.1. Closed Form Solution of the Problem

The closed form solution of the problem can be taken from standard literature [20]. The displacement on the free end for transverse load can be found by the equation 9.

$$\frac{d^2u}{dX} = \frac{M(X)}{E(X)I} \tag{9}$$

Whereas $M(X) = P(X - L)$ is moment

$I = \frac{bh^3}{12}$ is moment of inertia.

Since P, I and L are constant in our case thus the displacement will be

$$u = \int_0^L \int_0^X \frac{P(X-L)}{E(X)I} dX \tag{10}$$

$$u = \frac{P}{E_1} \int_0^L \int_0^{\frac{x}{L}} \frac{(xL-L)Ldx}{\gamma(x)I} \tag{11}$$

equation 11 is solved using the symbolic computation tool available in Matlab for different value of the power n, and value of E_1 & E_2. The value of displacement thus obtained will be used to compare convergence results of the FE analysis.

4. Comparison of the Convergence Characteristic of 'Material Mesh' Element with Geometric Mesh (Equal Length Element)

This section compares the convergence characteristic of material mesh and geometrical mesh. In both cases, the element is considered as graded.

4.1. Convergence comparison for constant E_2/E_1 (γ_{max}) ratio.

For constant γ_{max}, the convergence of the FE analysis was studied by varying n. The results for γ_{max}= 5 are shown in Figure 6. It is interesting to note that geometrical mesh is effective for n>1 whereas material mesh was effective for 0<n<1. The reason for this effect can be attributed to the effect of different mesh size and different γ_{max} values on the convergence. To further analyze the cause, it is considered necessary to see the effect of γ_{max} variation on the convergence characteristics.

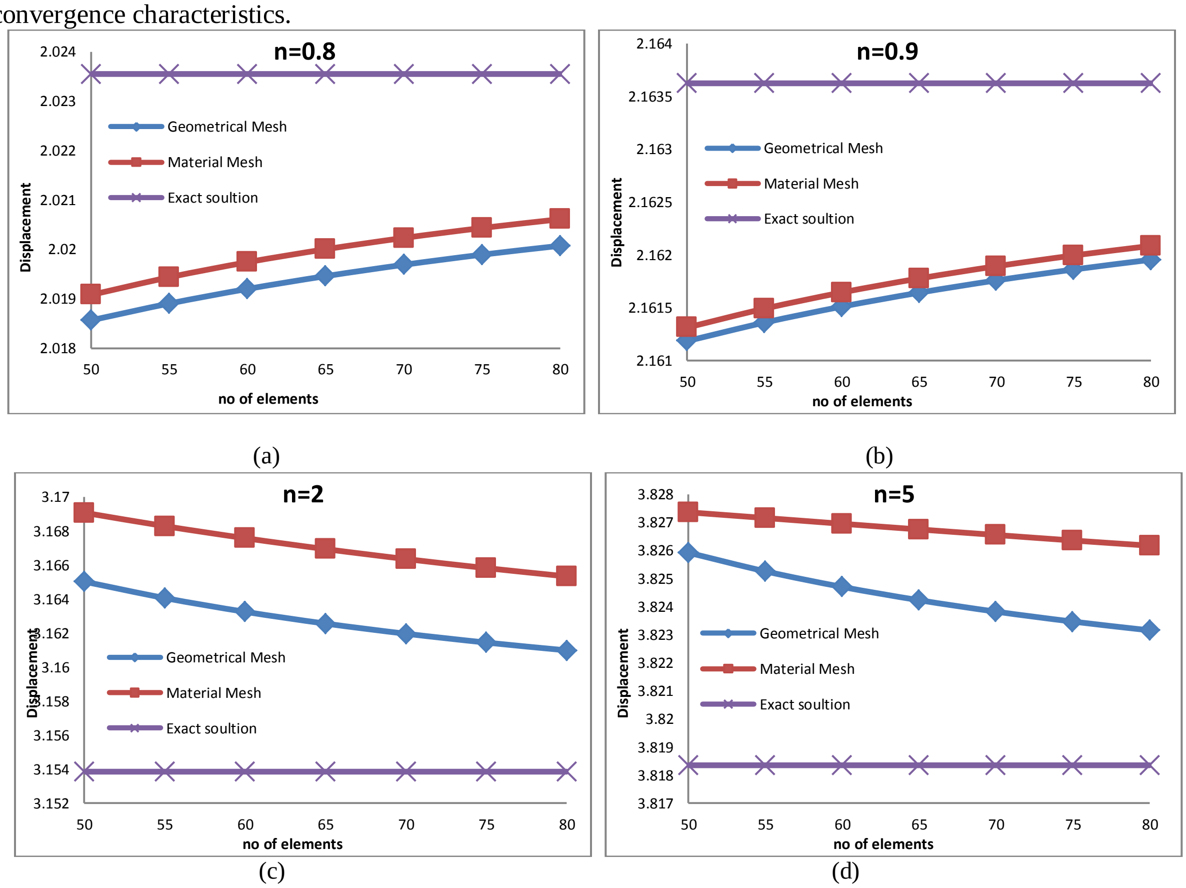

Figure 6: Displacement results (a) n=0.8. (b) n=0.9. (c) n=2. (d) n=5.

4.2. Effect of variation of γ_{max}

This simulation is similar to that done in Section 4.1, except that value of $\gamma_{max}$is also varied from 1 to 100; material mesh, geometrical mesh and exact solution were compared for different values of n. Some representative results are shown in Figure 7.

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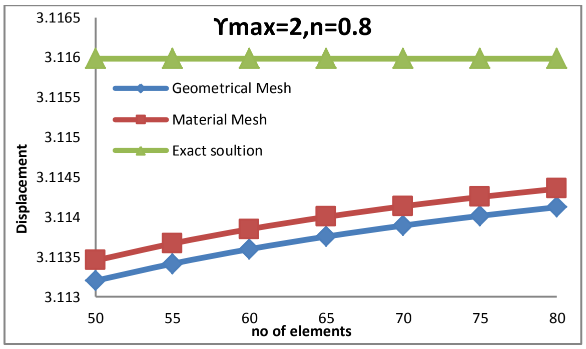
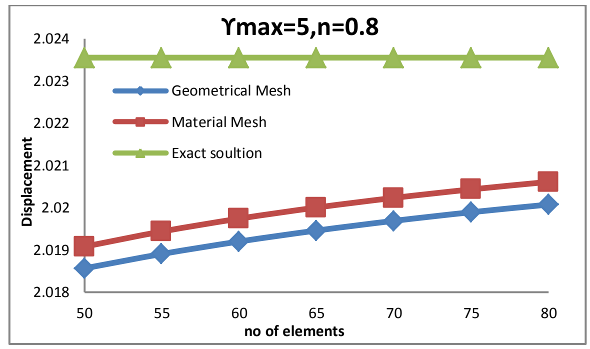
Vol.5 Issue.5, May 2017
Pg: -33-45



(a)



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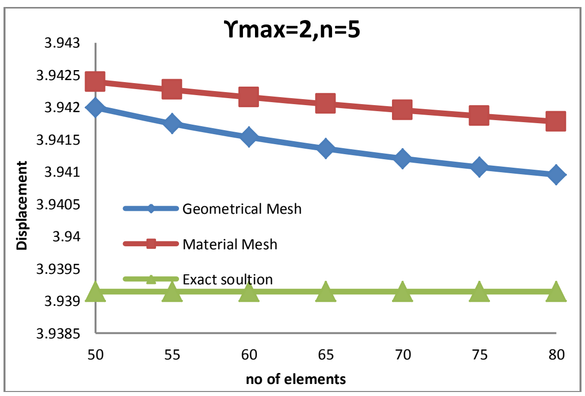
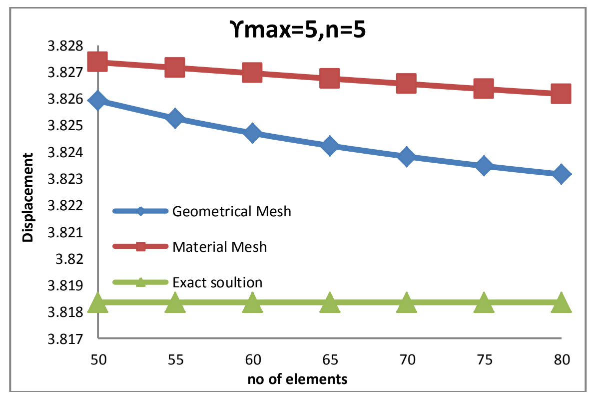


(c)



(d)

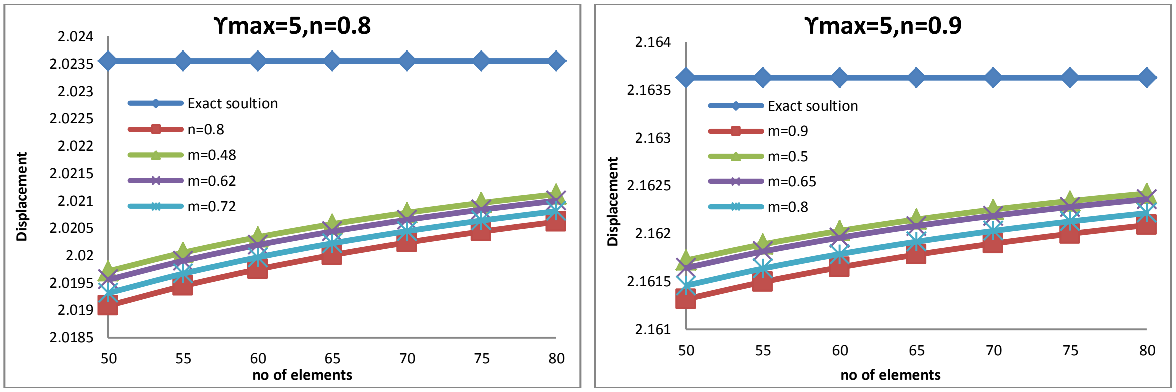
Figure 7: Displacement results for γ_{max} 2 & 5 (a) (b) n=0.8, Displacement results for γ_{max}2, & 5 (c) (d) n=5.

It seems that, for different γ_{max} ratios, the results obtained in both cases for 1<n<5 & 0<n<1 have the same pattern as given in Section 4.1. This analysis gives us a direction that probably meshing the element with the element with the same power 'n' needs to be revisited.

5. Taking Different Power Index for Material Meshing.

Section 4 also indicate that if we take the material mesh based on the power-law index n for material distribution, the convergence is general may not be optimal. Thus it makes sense to use some other power say 'm' which is in general different from 'n'.

For creating mesh, a power 'm' is used to create material mesh and the results in Figure 8 shows convergence result in between power-law index n versus m for constant γ_{max}. The value of m for best convergence is taken from the graph and termed as m_{opt}.



(a) (b)

Figure 8: Displacement results for γ_{max}= 5 (a) n=0.8 versus m=0.48, 0.62 & 0.72(b) n=0.9 versus m=0.5, 0.65 & and 0.8.

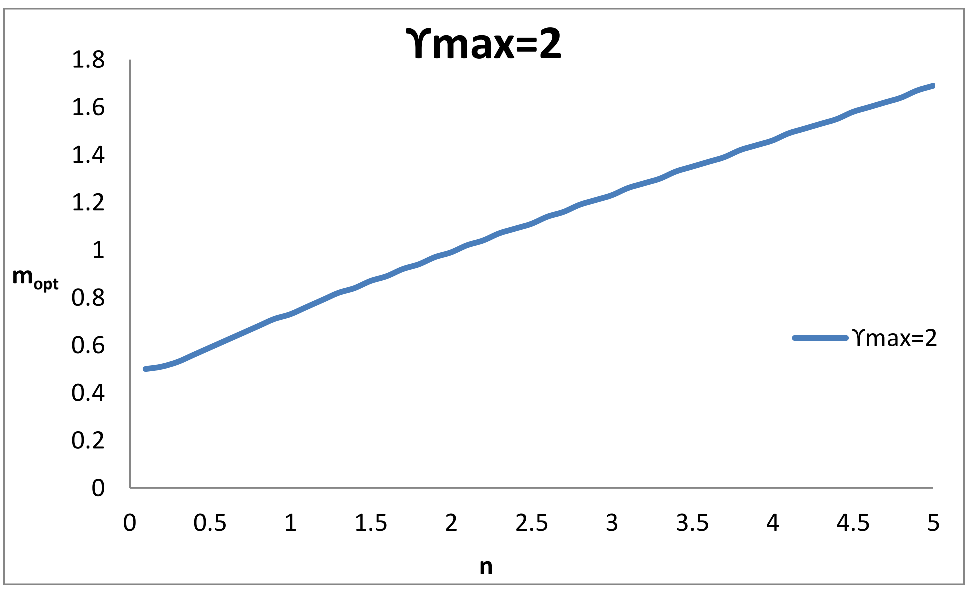


Figure 9: Actual powers used for defining FGMs *n* versus optimal power used *mopt*.

Figure 9: shows the plot between n and m_{opt} for γ_{max}=2. This plot is fairly linear and can be expressed as

$$m_{opt} = 0.24n + 0.49 \tag{12}$$

Similar numerical study is done for varying γ_{max} and it is shown in Figure 10.

Figure 10 shows a relationship between n and m_{opt} for different values of $\gamma_{\max}$. The nature of the graph is linear for n>1. And becomes non linear for large $\gamma_{\max}$ in the range of 0<n<1 indicates in for higher $\gamma_{\max}$values

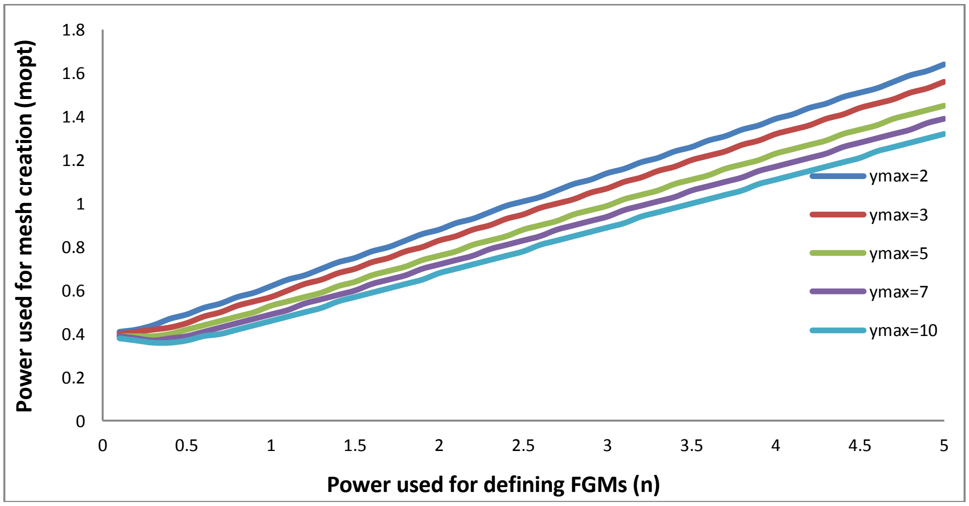
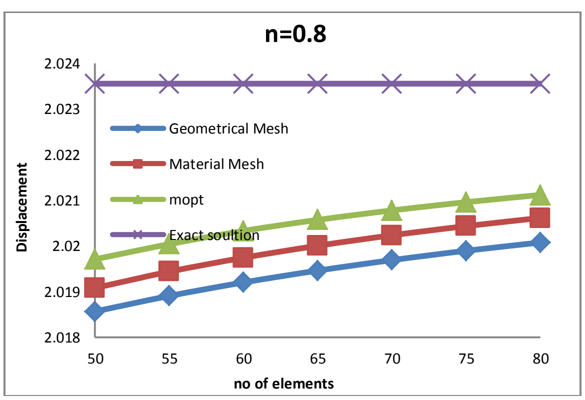


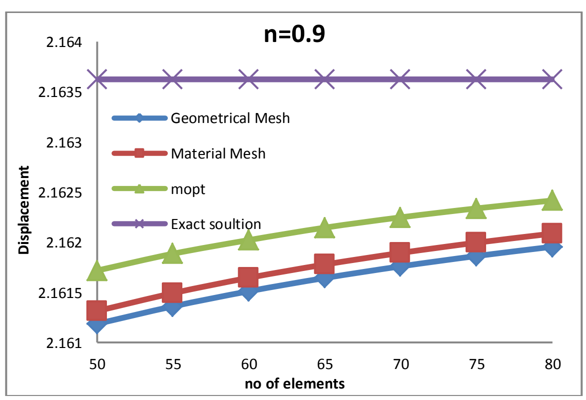
Figure 10: Relationship between powers used for defining FGMs (n) versus power used for mesh creation (m) for gradient mesh generation.

6. Results using optimum value of m (mopt) for meshing

In this section, we implement the meshing based on the power for meshing mopt as explained in Section 4. The result for different values of n and $\gamma_{\max} = 2$ are shown in Figure 11. All the results indicate that the convergence using material mesh is superior for all the cases. Similar results were optimal for higher volumes of $\gamma_{\max}$ but not shown here because space constraint. It was also shows that the convergence is now not affected by the power n, which is in contrast to Section 4.2 where we have different results n<1 and n>1.



(a)



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ISSN (ONLINE): 2321-3051
Vol.5 Issue.5, May 2017
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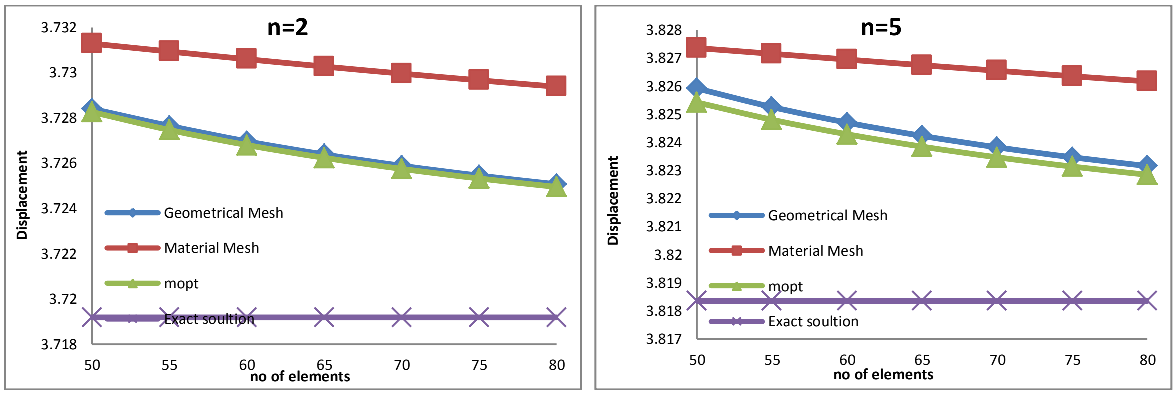
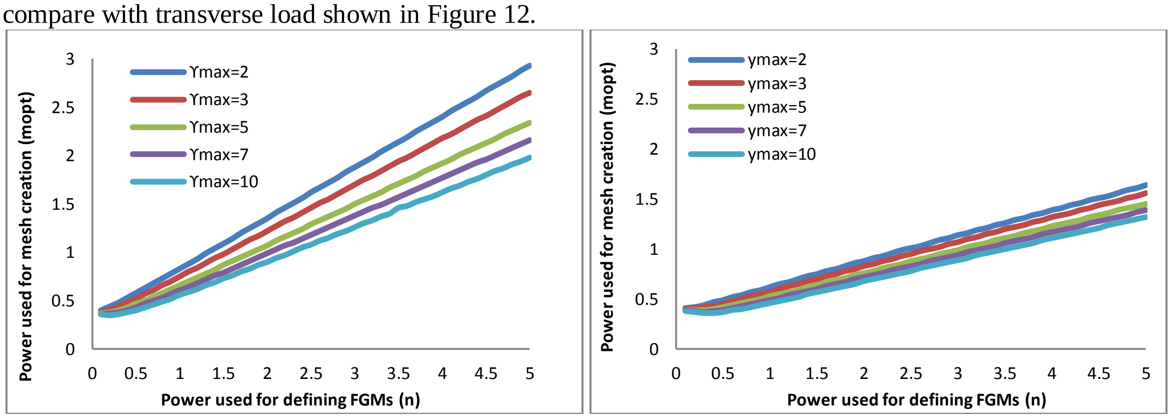


(c) (d)

Figure 11: Displacement results between geometrical mesh, material based gradient mesh, meshing based on the m m_{opt} (a) n=0.8 (b) n=0.9 (c) n=2 & (d) n=5.

7. Comparative Study between FGM Object with Transverse and axial Load.

In section 6 good convergence results obtain using relationship between elastic variation function and mesh size function for one dimensional FGM object in transverse load for varying elastic constant. Author follows same procedure for tensile load and obtains relationship between elastic variation function and mesh size function and compare with transverse load shown in Figure 12.



(a) (b)

Figure 12: Relationship between powers used for defining FGMs (n) versus power used for mesh creation (m) for gradient mesh generation.(a) tensile load (b) transverse load.

In the case of transverse loading the plot between n and m_{opt} for γ_{max}=2. This plot is fairly linear and can be expressed as:

$$m_{opt} = \frac{n}{4} + \frac{1}{2} \quad (13)$$

Where as in case of tensile loading the plot between n and m_{opt} for γ_{max}=2. Again this plot is fairly linear and can be expressed as:

$$m_{opt} = \frac{n}{2} + \frac{1}{3} \quad (14)$$

While doing a comparison with equation 13 & 14 the nature of equations is linear with different slopes and intersection.

8. Discussion

The goal of this study was to study the effect of the material based meshing on convergence of FE analysis results. Our study clearly indicate the superiors of the material based meshing vis a vis conventional meshing with mopt as a basis for meshing, the number of element for same acceptable error can be reduced to more than 50% in some cases, which is a huge computational advantage. While comparison with same case with tensile, linear function of curved is achieved.

9. Conclusion

Based on the simulation it can be concluded that material based graded element where the element size dependent on the power 'n' that defines material property gives faster convergence most of the cases.

It is also concluded that for material based graded meshing, it is recommended to use mopt for meshing in place of power n.

Though the above study is focused on one dimensional FE meshing can be extended for higher dimensional objects.

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